Quantum Interference Entanglement*

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Abstract

This experiment aimed to test the violation of the Bell inequality for the HVT theorem. And then show that no violation happens for the theory predicted by Quantum Mechanics. The experiment was conducted with great care and precision, following the instructions carefully to ensure accurate results. Despite conducting the experiment twice and multiple parts of it, the Bell inequality for the HVT theorem could not be violated. However, the Bell inequality for "some" state was confirmed in both Quantum Mechanics and HVT. We have reported that our $S = 0.12 \pm 0.05$. And as stated once, this result only confirms that the Bell inequality is holding for "some" entangled bipartite system (not maximally entangled but to some extent entangled) in HVT and Quantum Mechanics. This experiment can be quite finicky, and even minor discrepancies in the setup or alignment can significantly affect the results. The source of failure is unknown, but it is suspected that it may be due to wrongly tuned or calibrated angles or differences in the two BBOs in the system. Although the experiment was a failure, our team gained valuable insights into polarizers, optics, experiments that deal with optical tools, the general theory of Quantum mechanics, and entanglement.

I. INTRODUCTION TO QUANTUM ENTANGLEMENT AND INTERFERENCE EXPERIMENT

Entanglement has been one of the most counter-intuitive phenomena discovered in nature! The idea was first introduced into physics by the famous trio: Einstein, Podolsky, and Rosen (the name EPR comes from here!). It was a striking non-classical result of quantum mechanics! The idea is simple enough, yet hard to believe! Hence many theories and ideas were developed against this phenomenon. As Einstein described it as "spooky action at a distance!". Hence in 1935, the three good friends Einstein, Podolsky, and Rosen in an attempt against quantum mechanics developed and formed a theory in which they claimed that there are some missing and unconsidered variables in this quantum mechanics that arise from these counterintuitive properties in nature. Eventually, they published a paper which is now goes by the name of "Hidden variable theory" or from now on HVT for short! In 1964 John Bell showed that the "locality hypothesis" with HVT leads to a conflict with quantum mechanics. He postulated a mathematical theorem consisting of specific inequalities. The observation of a violation of these inequalities through experimentation would indicate the existence of nonlocality in states that are in favor of quantum mechanics and against the HVT! In this experiment, we went through the same procedure to violate Bell's inequality in the HVT theorem and confirm quantum mechanics.

* From now on we might call Quantum Interference Entanglement QIE for short.
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II. THEORY

In this section we will cover the working principles behind The general theories of quantum entanglement and interference experiment!

A. Entangled and unentangled states

The idea of entanglement is simple, yet it has complicated and counterintuitive consequences. A bipartite state that can not be written as a tensor product (Hilbert space product) is considered entangled. An example of an entangled state is:

$$|\psi_{EPR}>= \frac{1}{\sqrt{2}}(|H>_A|H>_B + |V>_A|V>_B) \quad (1)$$

And here is an unentangled state, where it is "separable":

$$|\psi>= \frac{1}{\sqrt{2}}(|H>_A|V>_B + |H>_A|H>_B) \quad (2)$$

Now for the pure idea of entanglement is usually evolved around the momentum or positions of the two states, or even the spin of a spin-1/2 fermion. But in this context and for the experimental advantages we will deal with the polarity of photons. Where "V" stands for vertically polarized and "H" stands for Horizontally polarized. Note that A and B stand for two different particles: A and B (in our case two photons!). Consider equation 1 which is a superposition state of both particles being either vertical or horizontal. If we know the state of particle A we spontaneously know the state of particle B as well. This non-factorability means the state of one particle that cannot be specified without making reference to the other particle. Such particles are said to be "entangled" and equation 1 is an entangled state Whereas in the equation 2 this is not the case. And there is no such an advantage! Now note that we can measure and use any rotated basis, so this means by multiplying the entangled state by a 2 by 2 rotation matrix with the argument of α we can measure and use a rotated basis. And our new basis are $|V_\alpha>$ and $|H_\alpha>$ So:

$$\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix} \begin{bmatrix} |V> \\ |H> \end{bmatrix} = \begin{bmatrix} cos\alpha|V> -sin\alpha|H> \\ sin\alpha|V> +cos\alpha|H> \end{bmatrix} =$$

$$= \begin{bmatrix} |V_\alpha> \\ |H_\alpha> \end{bmatrix}$$

Hence:

$$|V_\alpha>= cos\alpha|V> -sin\alpha|H>$$

$$|H_\alpha>= sin\alpha|V> +cos\alpha|H>$$

Where for example $|V_\alpha>$ describes a state with a rotated polarization by an Angel α from the vertical.In this basis, our $|\psi_{EPR}>$ will become:

$$|\psi_{EPR}>= \frac{1}{\sqrt{2}}(|V_\alpha>_A\,|V_\alpha>_B +|H_\alpha>_A\,|H_\alpha>_B$$

This state ($|\psi_{EPR}>$) in particular, the state with the normalization factor of $\frac{1}{\sqrt{2}}$ is called the maximally entangled state and it will put the entanglement effects at an extreme. even more it will make the calculation and procedures much more easier in such a way that the will reduce the number of some of the required parameters (makes the math easier). So producing this state is critical for this experiment and all steps and violation of the Bell's inequality depends on it (more on Bell's inequality in next sections).

B. Non-linear Optics and Down-conversion

In this experiment, we aim to generate entangled photon pairs using the process of spontaneous parametric downconversion (SPD). To achieve this, we will utilize Beta Barium Borate (BBO) crystals, which are an essential element in non-linear optics. The use of SPD to generate entangled photons has a rich history, dating back to its first use in testing a Bell inequality in 1988. However, for the purpose of our experiment, we will not delve into the intricacies of SPD and will focus on setting up our experimental configuration to observe entanglement phenomena. When a laser beam passes through the BBO crystals, a small fraction of the photons spontaneously decay into pairs of entangled photons through SPD (fig 1). These down-converted photons emerge at the same time and on opposite sides of the laser beam. To set up our experiment, we will use two BBO crystals with a 90-degree rotated optical axis from each other. This configuration enables us to down-convert vertically polarized photons from one crystal and horizontally polarized photons from the other crystal. The rule of photon conversion after passing through our discussed BBO system is as follows:

$$|H>_{incoming}\rightarrow e^{i\Delta}|V>_A\,|V>_B$$

$$|V>_{incoming}\rightarrow |H>_A\,|H>_B$$

Were an apparent relative phase difference of Δ is introduced in this down-conversion due to birefringence and dispersion. Now assuming that the initial blue pump light has its polarization described by $\sin\theta_l|V\rangle + \cos\theta_l e^{-i\delta}|H\rangle$, we can say that the down converted beam is now described as (δ is the initial phase!):

$$|\Psi_{DC}>= \sin\theta_l|V_\alpha>_A\,|V_\alpha>_B + \cos\theta_l e^{-i\phi}|H_\alpha>_A\,|H_\alpha>_B \quad (3)$$

where $\phi = \delta + \Delta$. Now note this critical fact that once we set $\theta_l = 45°$ and $\phi = 0°$ it is when we get our original $|psi>_E\ PR$ on equation 1.This indeed the maximally entangled state that we wish to work with!

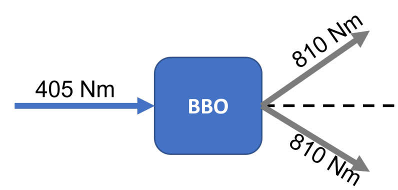

FIG. 1. A visual schem of what happens to the 405 nm laser after passing through a BBO

C. Probabilities

We are interested in finding some overlaps (probabilities) throughout this experiment. As an example and important probability in analysis would be:

$$P_{VV}(\alpha,\beta) = |<V_\alpha|_A <V_\beta|_B|\psi_{EPR}>|^2 \quad (4)$$

Where this expression evaluates to the probability of the vertically polarized photons in angle α basis and vertically polarized photons in β basis. By using the equation

1 and the equations we derived for $|V_\alpha>$, after the rigorous Braket algebra we can conclude that:

$$P_{VV}(\alpha,\beta) = \frac{1}{2}cos^2(\alpha-\beta) \quad (5)$$

Interestingly enough this probability dose not depend on individual basis angles but it depends on the difference of these two angles on this particular probability. Similarly other probabilities could be defined: $P_{VV}(\alpha,\beta), P_{HH}(\alpha,\beta), P_{HV}(\alpha,\beta), P_{VH}(\alpha,\beta)$. These probabilities are also useful in our theoretical analysis and could be evaluated similarly as outlined.

D. Parity of the polarization correlations

Let us define the parity of the polarization correlations in the basis α and β:

$$E(\alpha,\beta) = P_{VV}(\alpha,\beta)+P_{HH}(\alpha,\beta)-P_{HV}(\alpha,\beta)-P_{VH}(\alpha,\beta) \quad (6)$$

Where E ranges from -1 to +1 and it means that all photon coincidences have opposite polarization, to 1, meaning they all have the same polarization. The parities at specific angles will aid us later in violating the Bell's inequality. Note that each individual probability is obtainable as demonstrated for $P_{VV}(\alpha,\beta)$ as an example.

E. Experimental Methods to Find Parity

The method introduced for calculation of the parity is useful as far as theory goes. In action in the laboratory for those measurements of the desired probabilities we have a clever method. Let us denote the total number of coincidences of photon pairs (entangled photons) at angle α and β as $N(\alpha,\beta)$. And also let us denote $\alpha_\perp$ and $\beta_\perp$ as two angles perpendicular to the original basis angles, α and β. For each individual probability we can claim that:

$$P_{VV}(\alpha,\beta) = \frac{N(\alpha,\beta)}{N_{total}}$$

$$P_{HH}(\alpha,\beta) = \frac{N(\alpha_\perp,\beta_\perp)}{N_{total}}$$

$$P_{HV}(\alpha,\beta) = \frac{N(\alpha_\perp,\beta)}{N_{total}}$$

$$P_{VH}(\alpha,\beta) = \frac{N(\alpha,\beta_\perp)}{N_{total}}$$

Where:

$$N_{total} = N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha_\perp,\beta) + N(\alpha,\beta_\perp)$$

Combining all the above statements with equation 6, for $E(\alpha,\beta)$ we have:

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) - N(\alpha_\perp,\beta) - N(\alpha,\beta_\perp)}{N_{total}} \quad (7)$$

This is actually a powerful way to enable us determine the parity experimentally, which eventually leads to violation of the Bell's inequality.

F. Bell's Inequality (CHSH Inequality)

John Bell's work demonstrated that any theory which presupposes that properties are both local and well-defined prior to measurement must conform to certain constraints. What is fascinating is that quantum mechanics surpasses these Bell inequalities for HVT theories, signifying that nature operates in a manner that exceeds the confines of HVT. Consequently, this proves that no local realistic theory can precisely explain nature. But proves that Quantum mechanics is a fine potential candidate. One version of Bell's theorem is the CHSH inequality, named after Clauser, Horne, Shimony, and Holt. We can calculate a quantity called S, which is dependent on four distinct E measurements and determined by the angles α and β used to assess the polarization of each photon. For the HVT Bell's inequality signifies that $2 \geq |S|$, whereas quantum mechanics allows for values up to $2\sqrt{2} \simeq 2.8$ which means the quantum mechanics version of Bell's inequality is $2.8 \geq |S|$. If the system adheres to a local hidden variable theory, then the CHSH inequality limits S to 2. Nevertheless, quantum mechanics anticipates that particular quantum states of the two photons, as well as carefully selected angles, can generate S values as high as 2.8, but indeed greater than 2. Therefore, the ultimate objective of this experiment is to violate the CHSH inequality, thereby rejecting local hidden variable theories and verifying the validity of quantum mechanics (acquiring an S value between 2 and 2.8). The S value depends on 4 distinct E values:

$$S \equiv E(a,b) - E(a,b') + E(a',b) + E(a',b') \quad (8)$$

where a, and b are our original angles and a' and b' should be some other angles. Note that it is critical to choose a,b,a', and b' smartly. It turns out that the best result could be obtain by setting the angle $a = -45°$, $b = -22.5°$, $a' = 0°$, and $b' = 22.5°$. These four angles will have an extreme effect and will be the best choice for us. It worth to note that S depends on four values of E's. And then each E depends on four other probabilities. This means that S is dependent on 16 probabilities. In other words (if we want to think rather experimentally) 16 measurements is required in order to determine S value!

III. METHODS AND PROCEDURES

In this section the procedures and methods and experimental setups are outlined.

A. Setup

The central focus of this experiment is to generate a specific entangled quantum state of the polarization of two individual photons, a Bell state. This state can be represented as a superposition of both photons being either horizontally or vertically polarized. To achieve this, photons with a wavelength close to 405 nm are sent from a diode laser to a pair of non-linear beta barium borate (BBO) crystals. The crystals facilitate the spontaneous parametric downconversion process where a violet photon can decay into a pair of red photons with their polarization determined by the optical axes of the non-linear crystals. The photon pair emitted under a small angle is then detected after passing through polarization-manipulating optics using two Avalanche Photodiodes (APD). The detection of two APDs firing within a short time interval indicates that the observed events were indeed caused by a photon pair rather than stray light.
Now we will go over all the setup. Please refer to the fig.3 for a diagram or to the end of the report on fig.8 for an enlarged version. The initial phase of the experiment involves the utilization of a diode laser which has a maximum power output of approximately 120 mW at 405 nm. It is essential to note that this laser poses a potential hazard to the eyes at a wavelength of 405 nm(class 3B laser), and hence safety goggles were worn during the experiment. The diode laser is regulated by a Thorlabs LDC205C diode laser controller and observed by a Temperature Controller to ensure temperature control. It was ensured that the diode laser was in use at every stage of the experiment to maintain consistency.
A plot of power vs. varying current is included here for a better grasp on behaviour of the laser(fig.2).

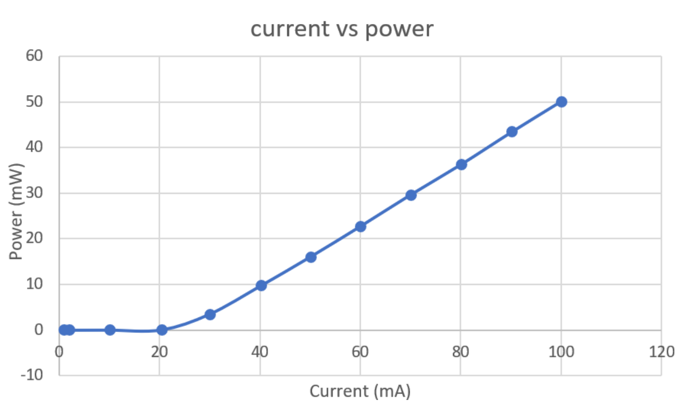

FIG. 2. Plot of the power of the 405 nm laser as a function of varying current. Note the rising trend and the stable trend along 0 to 20 mA.

Right after the isolator, there is a half-wave plate where we must set it up in such a way that it only allows the Vertically aligned photons. We can achieve that by trying to find the angle at which no light passes through and also once we flip the polarizer no light gets through as well. This ensures that the light to pass through is all vertically polarized.
Let us also Note the property of the half-wave plates which is each rotation by a certain degrees must be divided by the factor of 2 before the rotational operation. As an example a rotation of 45 degrees corresponds to a rotation of 22.5 degrees. And similarly a rotation of 90 degrees corresponds to a rotation of 45 degrees.

Next, we have to mirrors to direct the light into the next two half-wave plates labeled θ and ϕ. Then the light shines through the two-BBO system and splits into two branches! Each branch goes through an identical path. We call them photons A and Photons B. For instance, the photons go through another half-wave plate (plates α or β), these are actually the basis angles that we will change throughout the experiment.
After that, there is a Beam Splitter. It is basically a crystal that only transmits a certain type of polarization. For photons on path A, only the horizontally polarized photons, and for path B only vertically polarized photons are allowed. Then They pass through a low pass filter (for the sake of more accuracy and filtering out the unwanted photons), and then the fiber couplers will collect the photons. A fiber coupler is a device that collects the incoming photons and sends them to the avalanche photodiode (APD), which converts single photons into sizable (about 1 V) electronic pulses. They can detect anywhere from hundreds of photons to tens of millions of photons per second. The APDs are powered by a power supply that beeps if it overloads by bright light.
Then there is the field-programmable gate array (FPGA) that looks for signals from detector systems A and B within 5 ns of encounter. So if A and B fire photons within 5 ns of each other, our trustworthy FPGA will count that as a coincidence ("entanglement"!) and hence send it to the computer and get recorded. FPGA is also capable of recording the signals of A and B (those signals coming from detectors A and B within a time larger than 5 ns.). Note that throughout all the steps of this experiment it was tried to keep all the leads closed and turn all the excess lights (including room's main light) off. This will imporve the results significantly

B. Alignment

three Types of alignment must be done before the start of the experiment:

1. Violet Beam Path:
   The alignment procedure involves adjusting the direction of the violet laser beam to pass through the center of the BBO crystals and the center of the

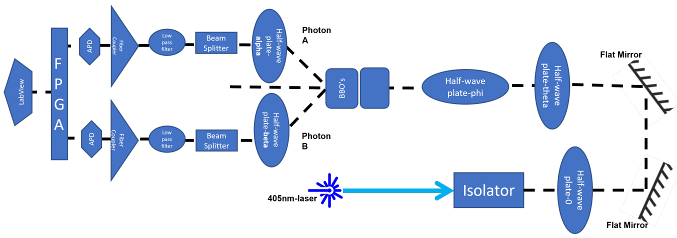

FIG. 3. A schematic of all the parts and circuitry of this experiment. An enlarged version of the diagram is provided at the end of the report.

paper target near the detectors. The circular paper target is used as a reference point for the first stage of the optics calibration. The first mirror is adjusted to center the beam on the target and the iris, while the second mirror is adjusted to center the laser beam on the center target near the detectors. Once the laser is aligned, the iris in front of the BBOs can be opened.

2. Infrared Beam Path:
The alignment process involves removing the low-pass filters and connecting the optical fibers to the infrared lasers. Then aiming the test laser precisely at the center of the BBOs and adjusting the detector height and angle until the red and blue laser spots overlap at the BBOs. Once the alignment is complete, the filters can be replaced on each detector, and the optical fibers can be reconnected to their corresponding APDs, making sure that the notch on the connector is lined up correctly. It is essential to be careful when removing the optical fibers from the APDs to avoid overloading them with ambient light, which can damage them.

3. Detection Arm Angle:
We adjusted the angle of each detection arm using the micrometers until we found the optimal angle that maximized AB coincidences count once the laser is on. Once we found the ideal setting, we took note of the exact readings, as it was not necessary to move the arms again.

C. Finding the optical Zero of the Half-wave Plates α and β

Maxwell's Devil has changed the optical zero points of our polarizers. So where it reads zero is not actually zero, but offset by a certain value that nobody knows (but the devil)! Our proposed method to find the real zero was to remove the low-pass filters. Then try to maximize the Coincidence count of AB once the laser is on, by rotating the angle α or β. We will get four maximum peaks. Either one of them is fine since they are degenerate (the two maximums on top and bottom of the polarizer are essentially the same reference and the other two are generated because these are half-wave plates and basically just repeated.). By following these procedures we were able to record the optical zero point of our polarizer. The values are:

$$\alpha_0 = 308^\circ \pm 1$$

$$\beta_0 = 331.8^\circ \pm 1$$

D. Tuning the Angle θ

As discussed in theory section, angle θ must be 45 degrees in order to produce the Bell state. This angle ensures that the normalization factors will become $\frac{1}{\sqrt{2}}$, just like equation 1. To do this, once again we do not know the optical zero point of the half-wave plate θ (Thanks to Maxwell's Devil) thus need to approach the problem differently. To achieve 45 degree angle we have done these series of actions:
First removed the half-wave plates α and β from the optical platform. Then we tried to maximize the coincidence count while the laser is on, by changing the half-wave's plate angle. Again we encounter 4 maximums, but either one of them is fine. Once the point was found, we put back both half-wave plates into the system. Then we tried to equalize the counts of A and B at $N(45^\circ, 45^\circ)$ and $N(0^\circ, 0^\circ)$. This means the ratio of members in our bipartite system is in fact equal, and thus the Normalization factor of $\frac{1}{\sqrt{2}}$ is achieved. The A count and B count do not need to be exactly the same. In fact if they are within a multiplicative factor of 2, it is still considered a nice equal proportion. As a sanity check that our states are in fact entangled. We played with angles α and β in such a way to see the dependency of one to another. Meaning that if you "decrease" one angle (decrease the proportion of a count) it will automatically put an increase to the counts of the other detector. This ensures our states is basically entangled an previous stages are potentially working fine. We report that the tuned angle θ is:

$$\theta_l = 180.5^\circ \pm 1$$

E. Tuning the Angle ϕ

Once again, We have to tune to the right angle to cancel out the the relevant phase due to the BBO system and the initial phase of the wave function before hitting the BBO System. We must set this half-Wave plate in such a way that it will "cancel" out the phase ϕ and make that unwanted phase equal to zero, so we can generate the Bell state. In order to tune in to the right angle we have done the following procedures:
After tuning to the right angle θ, we started to set the basis angles α and β to 45° to the same direction and

try to maximize the number of coincidence at that basis angle. Once this was done, we left angle α unchanged but set the β not only back to optical zero point, but also an extra 45° toward the other direction, and then minimized the number of coincidences by tuning the ϕ. By minimization it is meant to get the minimum correlation of A and B. As long as the ratio is about 1 to 5, the minimization was considered achieved! So basically the initial setup was a rotation of both angles by 45 clock-wise and must be maximized, and the second setup is now one of them 45 degrees clock-wise and the other basis counter-clock-wise and have to be minimized. This was a smart trick and here thanks and shout out to Winthrop for providing this procedure. We report that:

$$\phi = 335^\circ \pm 1$$

F. Bell State Sanity Check

Ideally after the previous steps, we are now at the Bell state! We have to test and confirm this! To do this we followed the instructions listed on a paper written by D. Dehlinger and M. W. Mitchell [2]. We have done two sanity checks recommended. And here is where we got unexpected results. Unfortunately, both sanity checks failed at this stage. As Professor Holzapfel said about this experiment:"... you could have done everything correctly and yet you might not get the result you desired in this lab...".
Anyhow! Here is the procedure for sanity checks: We fix α to 0, 45, 90, 135 degrees and record the coincidence counts over a period of 10 seconds time window and as a function of varying angle β (did this for each fixed α). We generated 4 plots and the plots and how they failed us will be reported in the Analysis section.
The other check was to use the provided model by D. Dehlinger and M. W. Mitchell [2] to estimate the true experimentally evaluated values of our angles θ and ϕ based on that established model. The required data for this check was just the number of coincidences at 4 points, which are: $N(0^\circ, 0^\circ)$, $N(45^\circ, 45^\circ)$, $N(90^\circ, 90^\circ)$, and $N(0^\circ, 90^\circ)$. Once again, we will report the values and how these failed us as well in the analysis section.

G. 16 parity Measurements to Find S

The previous part was discouraging. Nevertheless, we still had hopes so in an attempt we tried to measure the S values, even though our Bell state was problematic! So using the equations 7 and 8 and the preferred angles: $a = -45^\circ$, $b = -22.5^\circ$, $a' = 0^\circ$, and $b' = 22.5^\circ$. , we measured the coincidence values over a period of 10 Seconds. We recorded these values and then we calculated the S value. This will be reported in the analysis section and will be discussed.

IV. DATA ANALYSIS AND RESULTS

Here we are releasing the whole sets of data and performed analysis done on those previous outlined experiments. You can also find a more in depth discussion of the covered topics in methods and procedure section here as well.

A. Bell State Confirmation! Reality or Not?

Here we will provide a more in-depth discussion about the sanity checks we have done for our Bell state. We will also provide the results and how we failed.
Our acquired plots for the Bell state (fig4 and 5) are not in agreement with the established Bell state plots from D. Dehlinger and M. W. Mitchell [2] (fig6). As it appears the plot in fig.4 is rather in relative agreement (although the phase difference between 0 and 45 degrees seems too low). But on the fig.5 the plots are totally off the model and there seems to be a vertical offset due to some unknown source. We are not sure what caused this offset as we definitely turned all the unnecessary lights off and had the lids of the boxes on throughout the data acquisition process. This unexplainable behavior made us certain that our Bell state is not accomplished (for whatever reason, and we could not yet figure this out).
There is a model introduced in the D. Dehlinger and M. W. Mitchell [2] which depends on only 4 specified coincidences (after a cure-fitting on those plots), and by using that we can find the actual angle θ and ϕ that were tuned in on our setup. Please refer to fig.7 to see our raw measurement results for these 4 coincidences. The model is as follows for θ and ϕ:

$$C = N(0^\circ, 90^\circ)$$

$$A = N(0^\circ, 0^\circ) + N(90^\circ, 90^\circ) - 2C$$

$$tan^2(\theta) = \frac{N(90^\circ, 90^\circ) - C}{N(0^\circ, 0^\circ) - C}$$

$$cos(\phi) = \frac{1}{sin(2\theta)}(\frac{4N(45^\circ, 45^\circ) - 4C}{A} - 1)$$

Once our data were input into this model we had:

$$\theta = 45.96^\circ \pm 1^\circ$$

This suggests our angle θ was tuned in nearly perfectly (it is so close to 45 degrees with a percent difference of lower than 3 percent).
But the problem appears to be on angle ϕ reported value by the model, which is:

$$\phi = 1.09i$$

As to our disappointment and wonder,This answer is not obviously physical (imaginary numbers came into play since there is an *arccos* function and the argument of this function turns out to be greater than one!). This provides information about our angle ϕ that was not tuned in correctly, and something prevented this that we are unaware of. We have repeated the experiment several times and each time we got an imaginary number which suggests our state is not tuned into the maximally entangled state. What is the cause? we are still uncertain. At this point, since we know this state is not definitely the Bell state it is useless to fit the model on our data and to perform a curve of best-fit analysis.

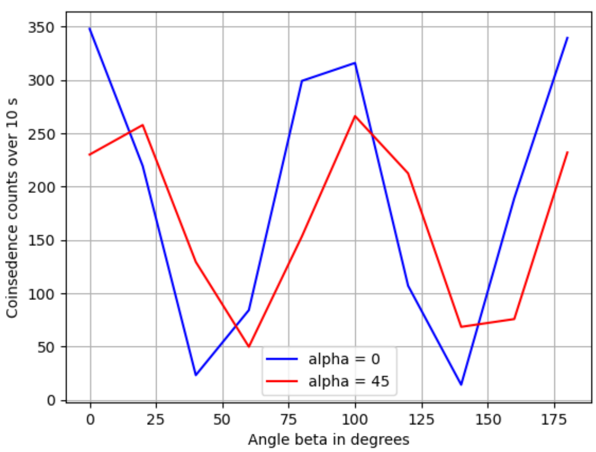

FIG. 4. This is the plot of our Bell state. It is in a relative agreement with the model, yet it deviates a bit as well.

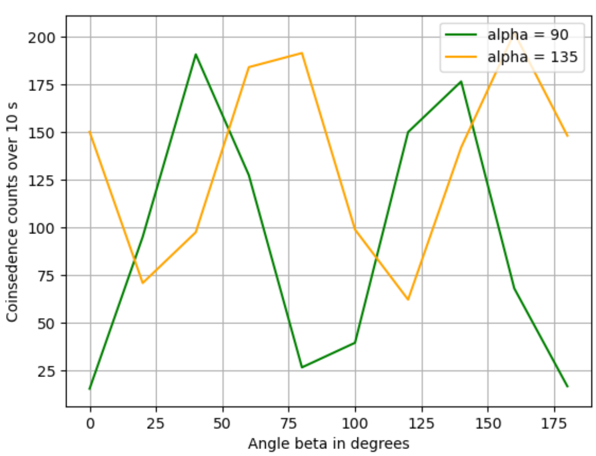

FIG. 5. This is the plot of our Bell state. The 90 degrees plot has an unexplainable offset from 135 degrees. What could be the cause of this offset?

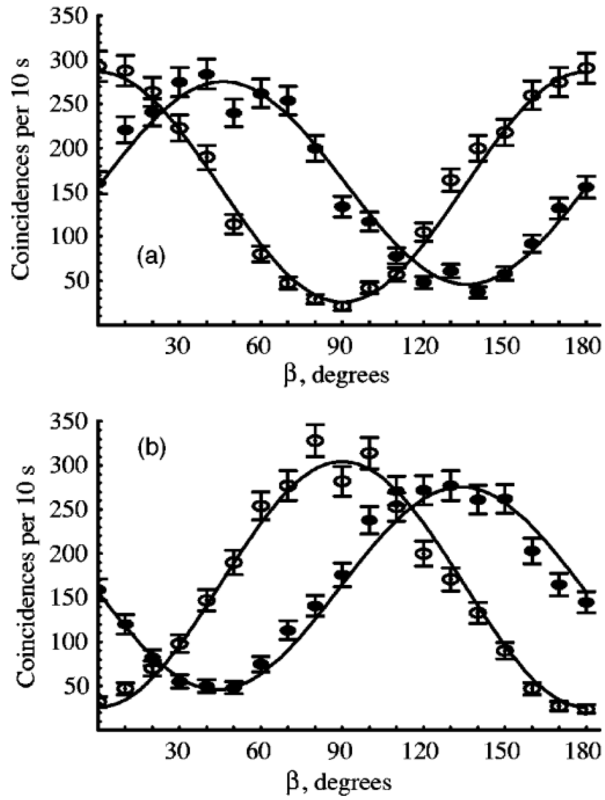

FIG. 6. This is the plot that D.Dehlinger and M.W.Mitchell [2] reported for their Bell state. Our plot deviate a lot from this plot.

\alpha	\beta	coinsedence number over 10 s
90	90	234
45	45	845.7
0	90	81
0	0	921.4

FIG. 7. A table of our 4 measurements for this model is provided here. This measurements lead to a fine result for angle θ but a poor and nonphysical one for ϕ

B. S calculation, and Bell state Violation! Fact or Myth?

After taking the 16 coincidence measurements and processing them as instructed on procedures and methods section, we came to a conclusion that our S value is:

$$S = 0.12 \pm 0.05$$

with four individual parity measurements of 0.086, -0.040, -0.101, 0.0926 (all ± 0.05). The parity values are within the expected range(-1 to 1) but seems to be too small to yeild a number on oredr of magnitude of 2. This result is not violating the Bell inequality. In fact, it is confirming it. Since it appears that we have not tuned into the Bell state properly we can not get to the extreme case of the S value (which is between 2 and 2.8) to violate this inequality for the HVT theorem. We even did the whole experiment one more time, but yet we got no better S value! Sigh!

V. CONCLUSION

To conclude, we conducted the experiment with great care and precision, following the instructions carefully to ensure accurate results. We did the experiment twice, and even some parts of the experiment multiple times. However, despite our best efforts, we could not violate the Bell inequality for the HVT theorem. On the bright side, we were just able to confirm Bell inequality for "some" state in both Quantum Mechanics and HVT. As we know, this experiment can be quite finicky, and even minor discrepancies in the setup or alignment can significantly affect the results (as Professor Holzapfel once said that: " ... you could have done everything correctly and yet you might not get the result you desired in this lab… "). The source of failure is unknown. We are suspecting that this might be due to the wrongly tuned-in or calibrated angles. Or it could have been due to the differences in the 2 BBOs in the 2 BBOs system. Where the imperfections of not having the exact identical BBOs will potentially interfere with angle tunings. We were close but we accept this defeat! Physics won't be just repeated success, it also includes a lot of failures, and we have learned that today! Nevertheless, we have not lost hope! In fact, we gained valuable insights into polarizers, optics, experiments that deal with optical tools, the general theory of Quantum mechanics, and entanglement that otherwise would have been impossible to gain without going through this failure. We will not give up and take this as an opportunity that honed our skills.

VI. REFERENCES

QIE - Quantum Entanglement and Interference Physics 111B: Advanced Experimentation Laboratory University of California, Berkeley [1]

D. Dehlinger and M.W. Mitchell, "Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory". [2]

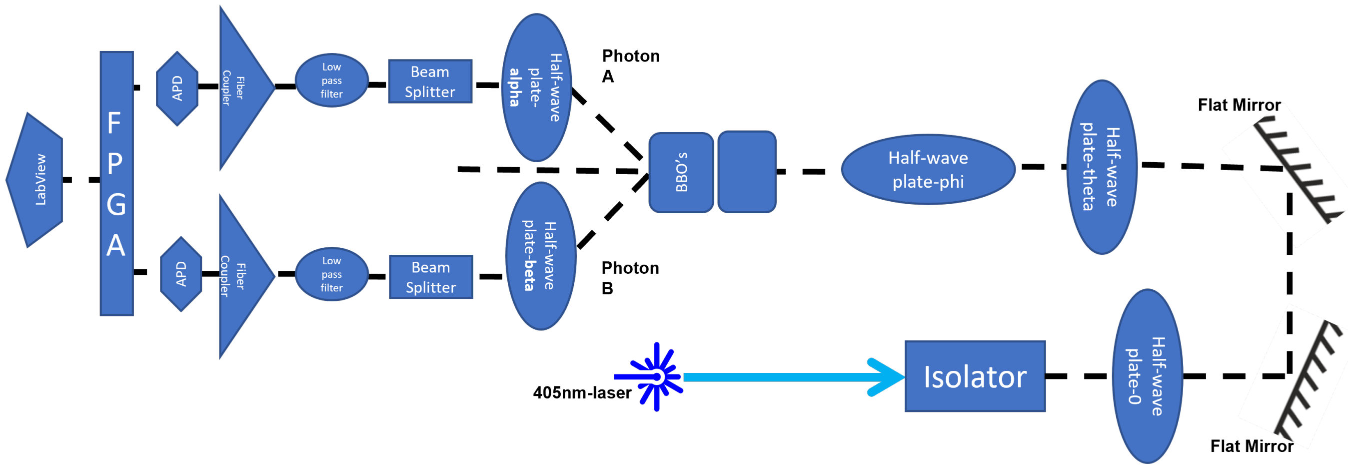

FIG. 8. For the sake of the importance of the diagram, an enlarged version of the fig.3 is placed here