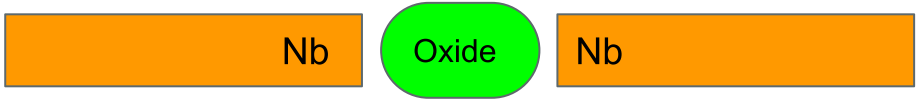
The Josephson Junction Effect

111B lab

By Kazem Ayat

Advanced Experimental Physics!



About JOS!

- . In 1962, Brian Josephson predicted that electron pairs could tunnel without resistance through an insulating barrier between two superconductors.
- When there is a voltage V across the junction, the supercurrents flowing through it oscillate at a frequency given very precisely by the relation $f = 2e V/h$.
- the basis for a new determination of the fundamental constant ratio e/h. (John Clarke) (better results!)
- This is a direct consequence of the macroscopic quantum state which exists in a superconductor

About JOS!

- A DC current can go through the junction with no potential difference, but when a DC voltage is applied together with a small alternating voltage, the I-V curve shows a characteristic step structure.

- Then why JOS to find e/h? Since the Josephson value of e/hk is believed to contain no quantum electrodynamic corrections

Application!

Josephson junctions have been used in a number of

Devices for example:

1) for the measurement of very tiny magnetic fields and voltages

2) the detection and generation of microwave and submillimeter radiation,

3) and of course the determination of e/h. Which has important consequences in physics

Objectives!

- Find 2e/h experimentally!
- Understand what is superconductivity and how it relates to the Josephson Junction effect.
- Explore and understand what are the DC and AC Josephson effect.

Theory!

Theory!

Cooper Pairs are important!

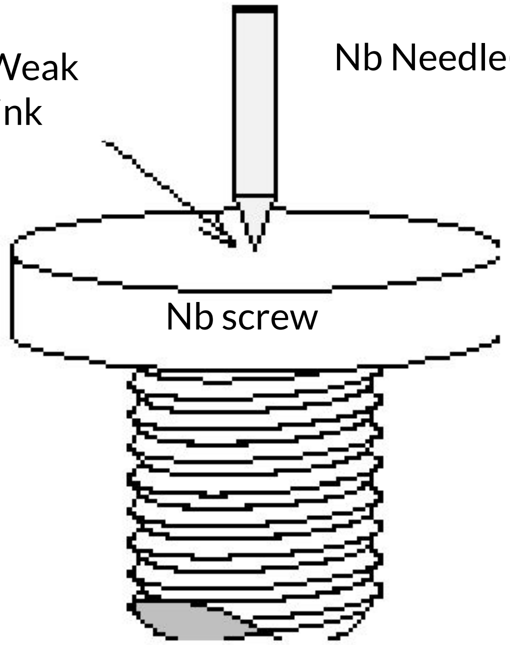
When a metal becomes superconducting at low temperatures, some of the free electrons become paired together.[2,16] The electron–electron attraction producing this pairing arises from the electron–phonon interaction, which is required to be greater than the Coulomb repulsion between the electrons in order for the metal to become superconducting.

Theory

Please refer to theory sheet below!! (Pg 50)

Procedure

Point Contact Junction :



The central component of this experiment is the point-contact junction (PCJ)

Making the needles!

6 inches of Nb wires

Used pen vice

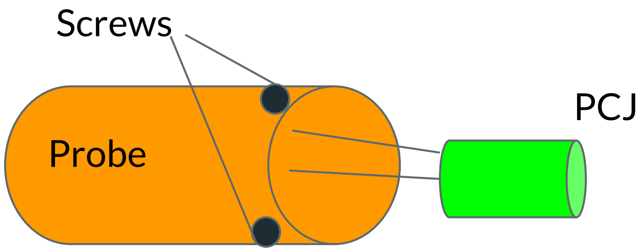
About 45 degree
with Horizontal

Smooth with fine sandpaper

Let the tip of the needle Oxidize over night!

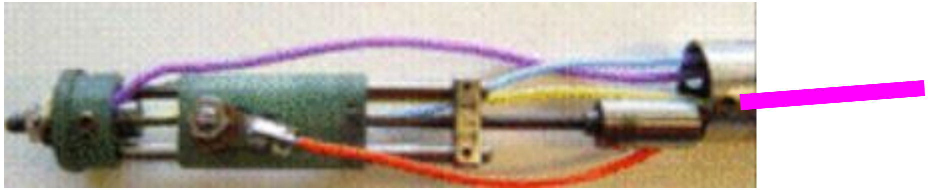
We made more tips! We broke the tip once! So we used our second tip!

Remove the junction assembly from the probe



The screw must rotate CW to open!
CCW to make tight!
Very counter intuitive!

Also a rotational movement of the Allen wrench is required as well



here!

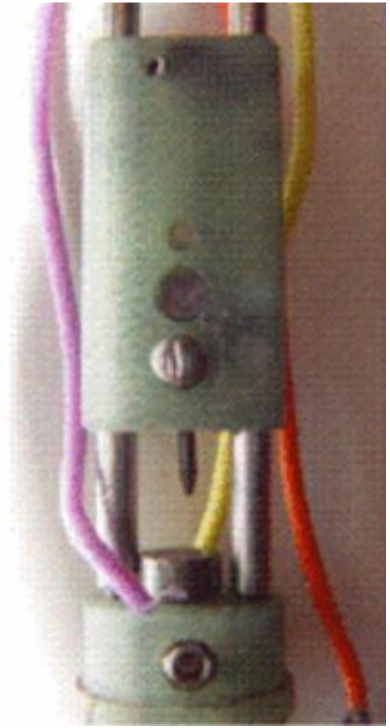
Installing a point in the assembly

1. First back off the differential screw (D) until it comes out of the housing.

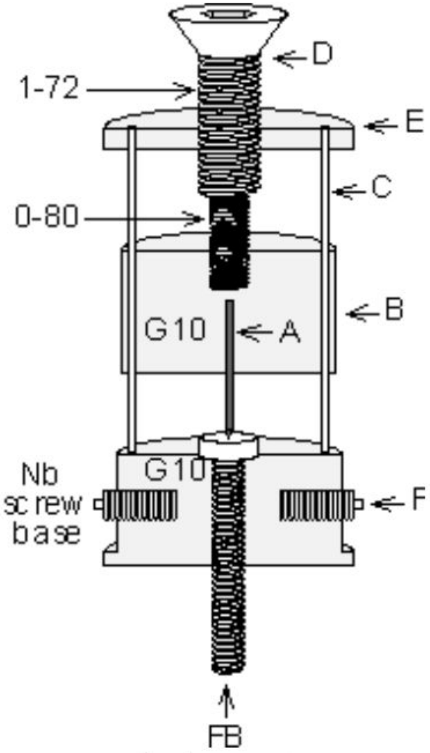
2. Loosen the nuts associated with the screws which have solder lugs attached to them, in piece (G10). don't need to remove the nuts entirely (and it's easier).

3.Using small needle-nose pliers, gently pull the needle out of the housing (B). note to prevent the screws from falling off by placing two fingers on opposite sides of the assembly

4. Throw the old needle away



e differential screw



Installing a point in the assembly and putting it back into the probe!

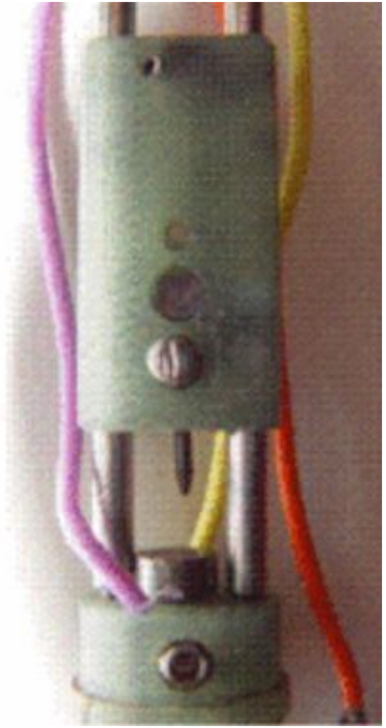
5. Use light sandpaper to remove the oxide layer from the back end of the point

6. Thread the new point through the lower screw shaft

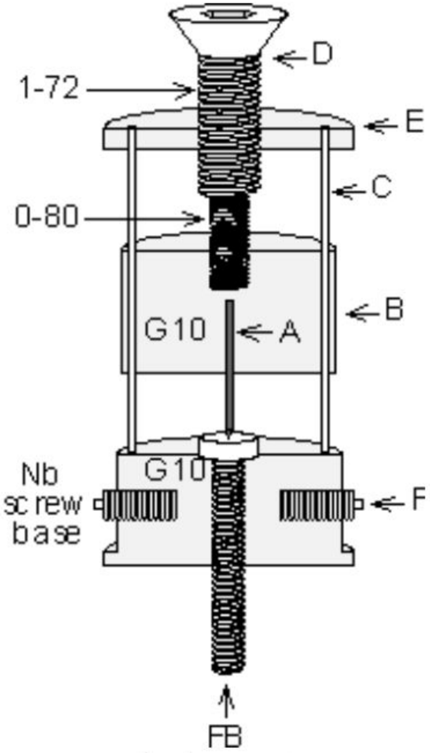
7. Now tight the side screws using needle-nose pliers

8. Put back the diff. Screw , and tight it but clearly make sure the needle's tip is not touching. Leave 5mm of space.

9. Make sure that the rod inside the probe gets into the head of differential screw and won't come off. Be wary of wires. They might get destroyed



e differential screw

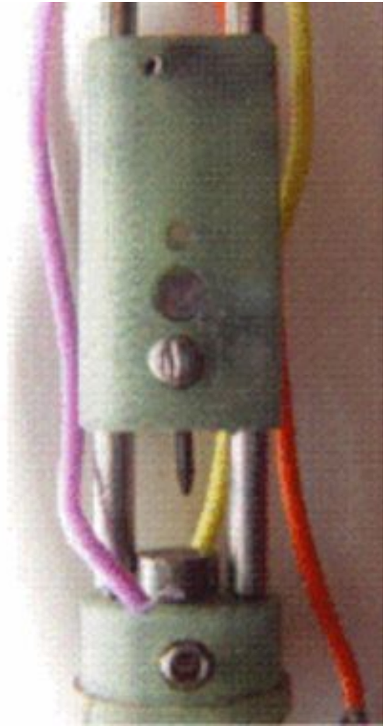


Installing a point in the assembly and putting it back into the probe!

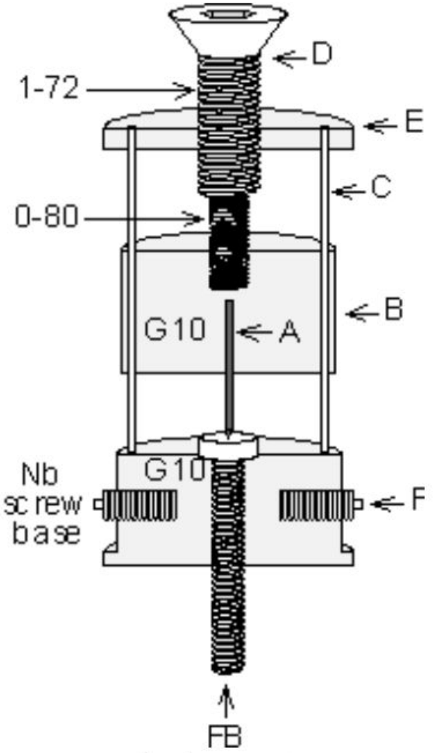
10. Slowly insert the junction into the tube

11. Now there should not be any resistance, everything should go smooth!

12. Now tight the side side screws on the probe. Not It must turn CCW!



e differential screw



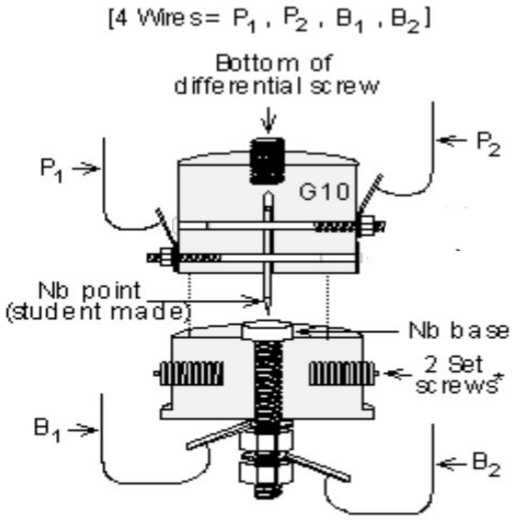
Check the Connections!

These connections must be checked before advancing:

Infinite Resistance:
- B1-GND
- B2-GND
- P1-GND
- P2-GND

Some Resistance:
- B1-B2
- P1-P2

Infinite Resistance:
- P1-B1
- P2-B1
- P1-B2
- P2-B2



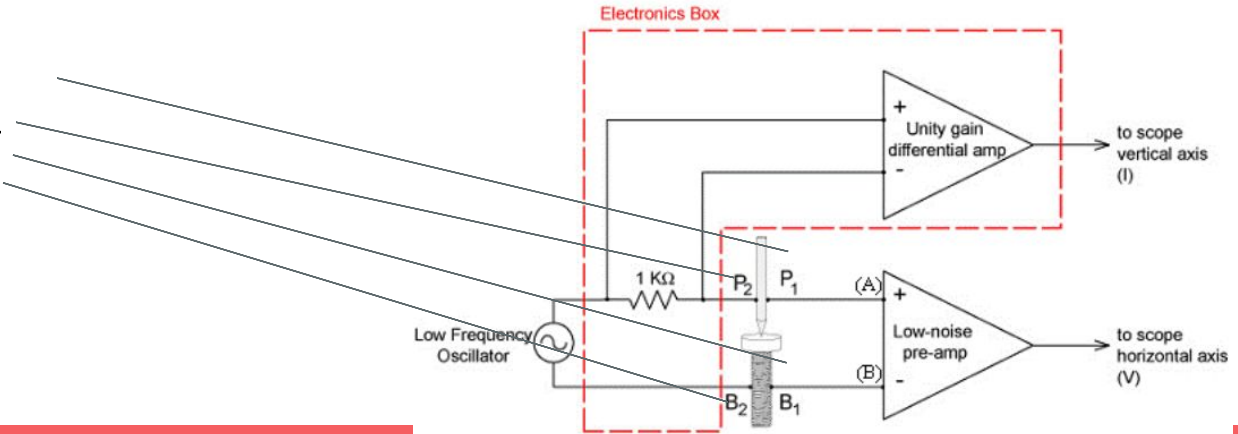
Note: (scale is 50 ohms, and infinity relative to this scale!)

4 Wire Measurement!

Note that since the impedance of the junction is small, We are using a standard **four-terminal measurement**. This is a very important way of measurement for this experiment. We will get a resistive measurement instead of a I/V curve measurement.

Ideal for measuring low impedance, the four-terminal measurement technique applies current through one pair of leads and measures voltage on the other pair, removing the “series-lead” impedance error completely.

4 wires configuration!

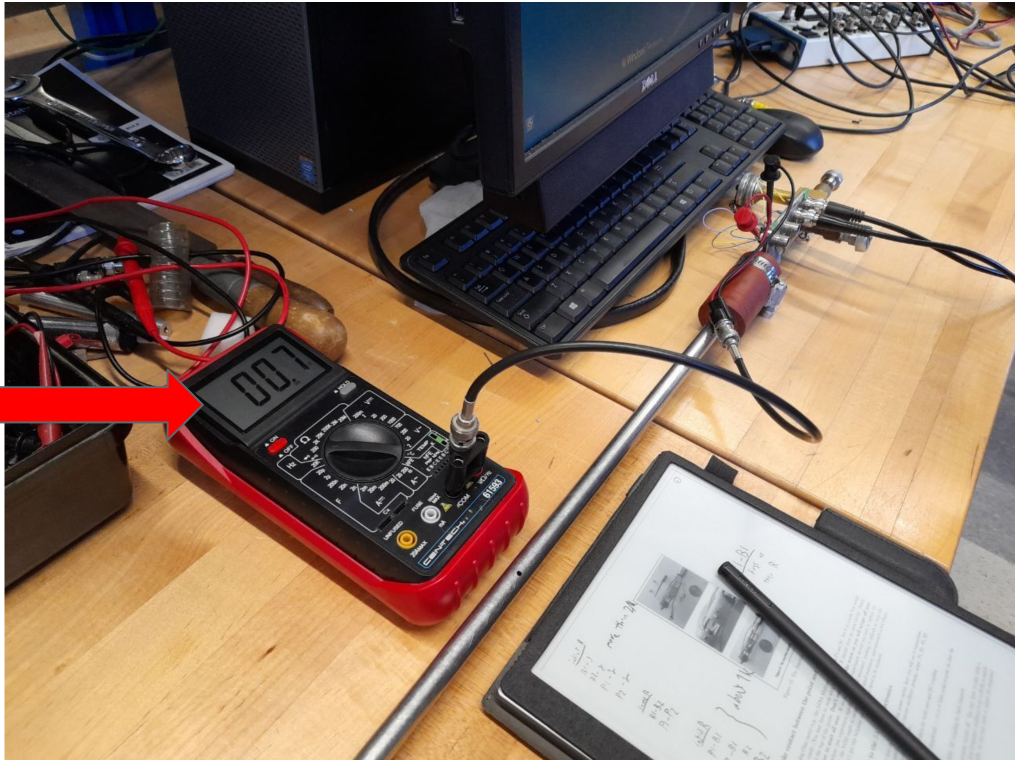


P1-B1 Connection!

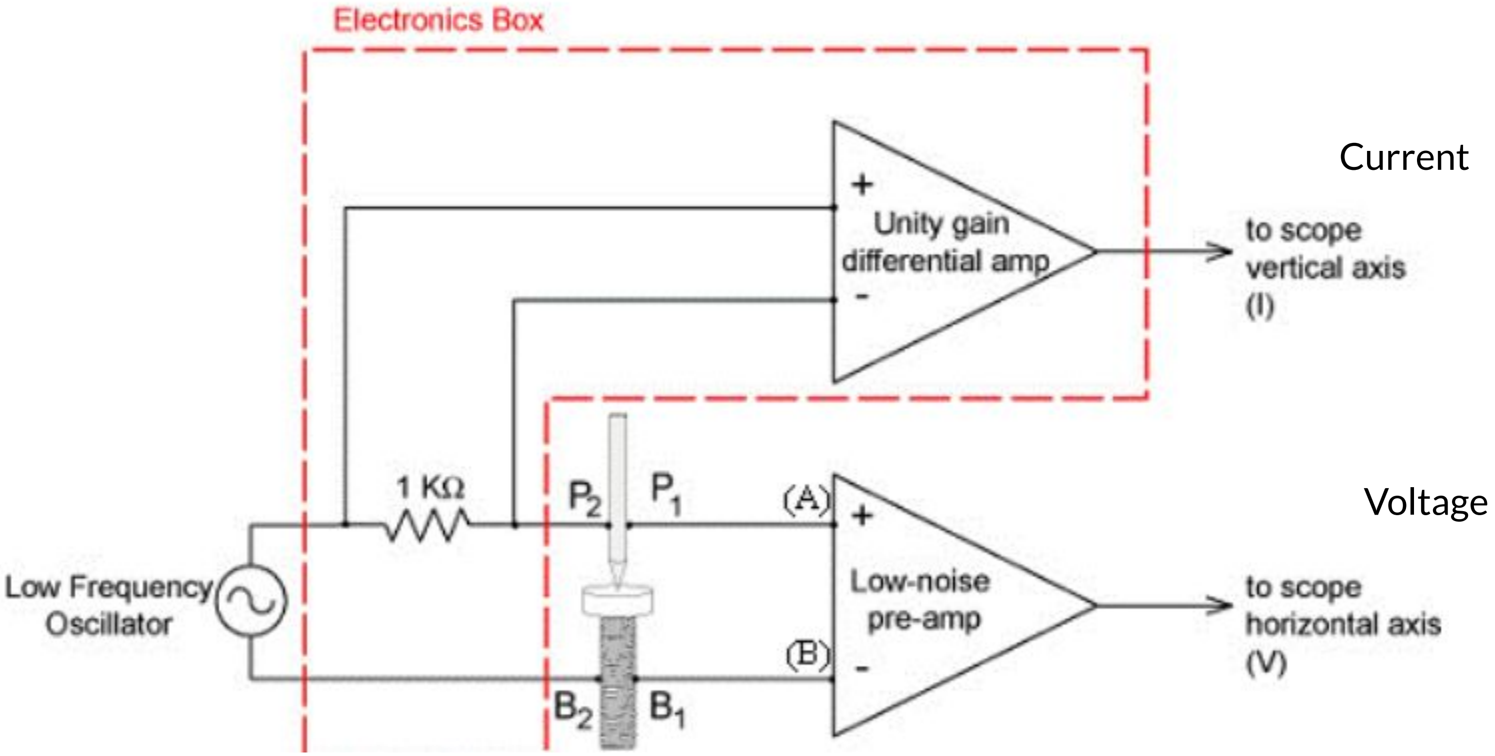
Now we connect the P1-B1 to our DMM and very slowly tight the differential screw! At all the time P1-B1 must show infinite resistance (scale is 50 ohms, and infinity relative to this scale!)

Suddenly at a sweet spot, the resistance turns really close to zero! In our case 0.7 Ohms!

This is the sweet spot, and we almost ready insert our probe into cryostat!



Electronic Diagram



Set up the scope!

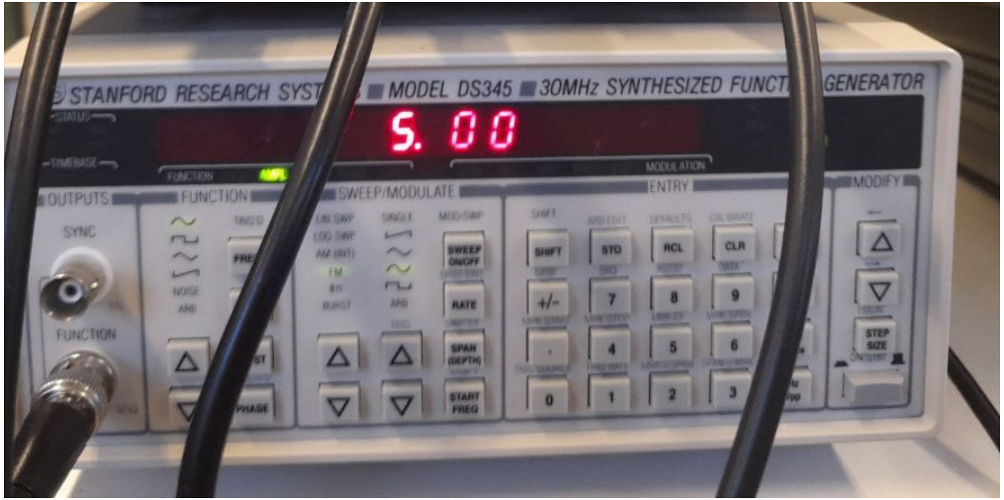
Set the oscilloscope setting to:

- X-Y mode
- DC coupling
- Divisions are not really important at the moment; since we'll change them a lot later

Low Frequency Oscillator:

Set the Low frequency oscillator to:

- Freq = 60 Hz
- Amplitude = 5 (peak to peak)

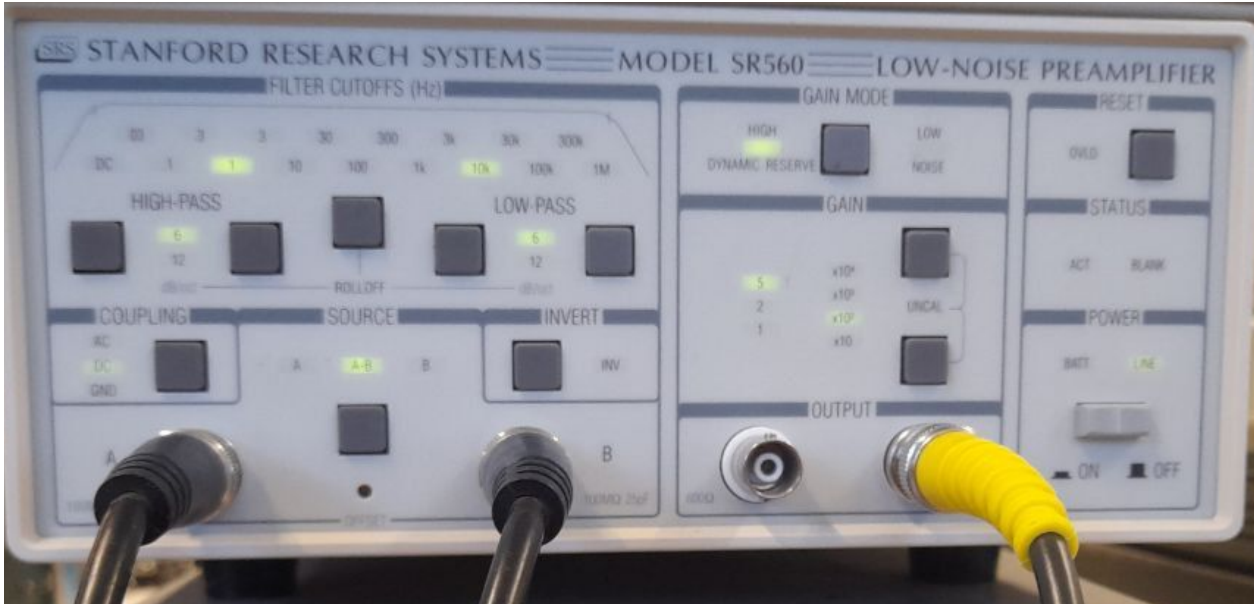


Preamp:

Set the preamp to:

- Gain =500
- DC coupling for both channels
- And bandwidth of 0.03 Hz to 30 kHz

but note that this eventually changes, and usually a narrower bandwidth is required. Since an overload in preamp happens if the bandwidth is too wide!



Know the depth of Liquid He

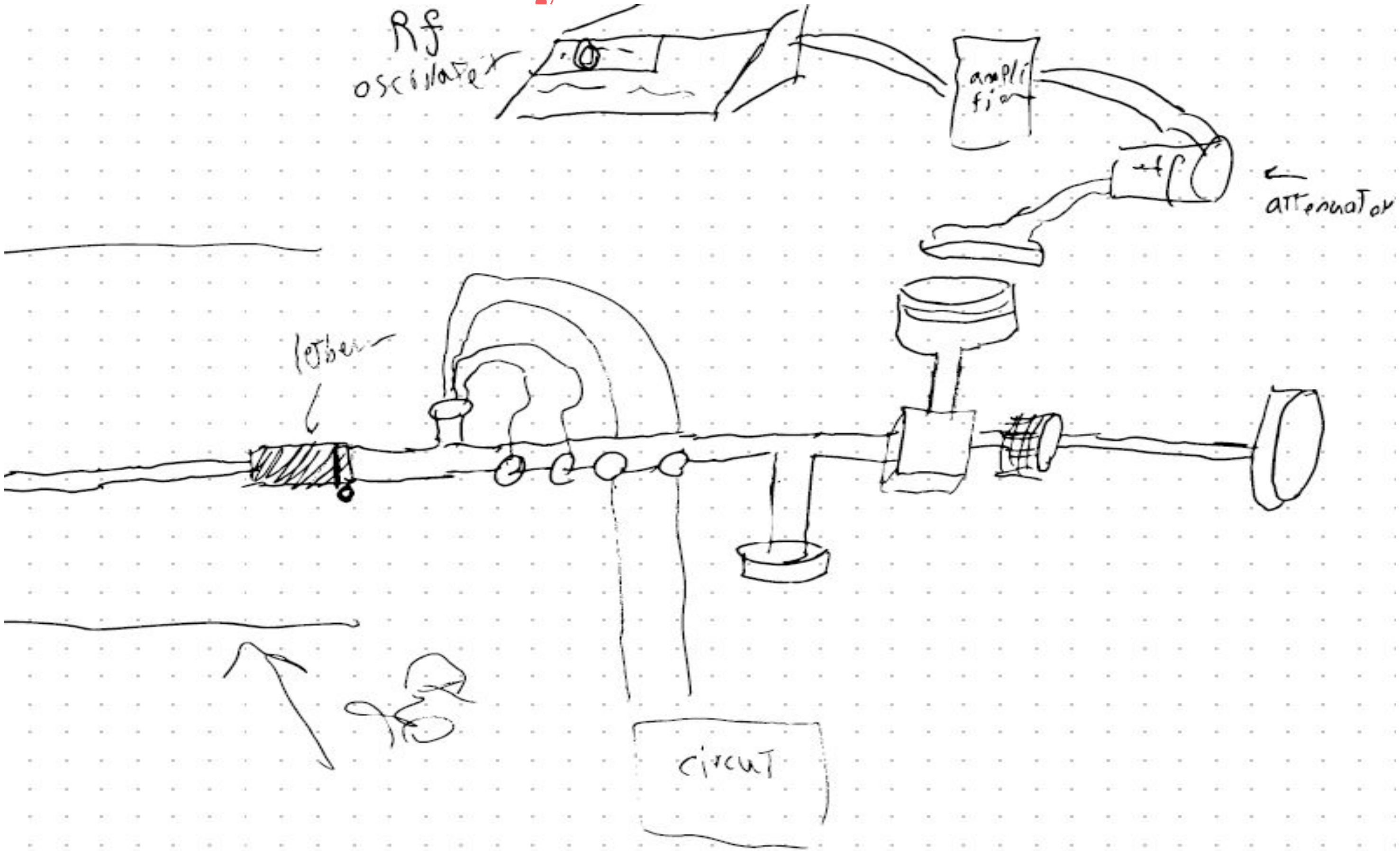
Feel the vibration!

Insert the probe into the Cryostat!

Now by practicing the safety warnings, we inserted the probe into cryostat!

Then We have all the connections to P1,P2,B1,B2 all ready and connected to the probe!!

Insert the probe into the Cryostat!



DC Effect!

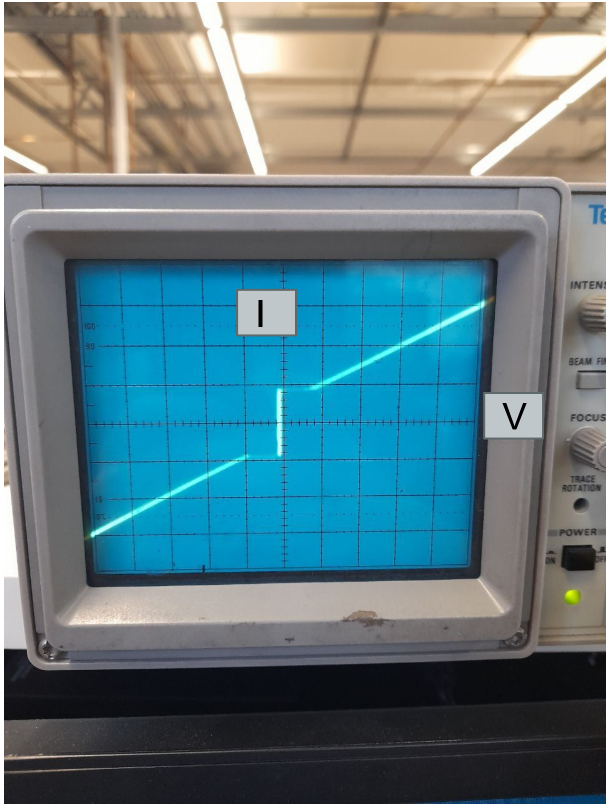
- Here now we should do adjust the probe's end! Very slooooowwwwly!

- Also Make sure preamp is not showing an overload! If yes, then widen the range!

- Make sure to not destroy the tip! By making it touching the screw too hard!

We test that with a magnet!
To make sure it's not a fake effect!
Noting the fluctuations!

We Actually broke it once!   :(

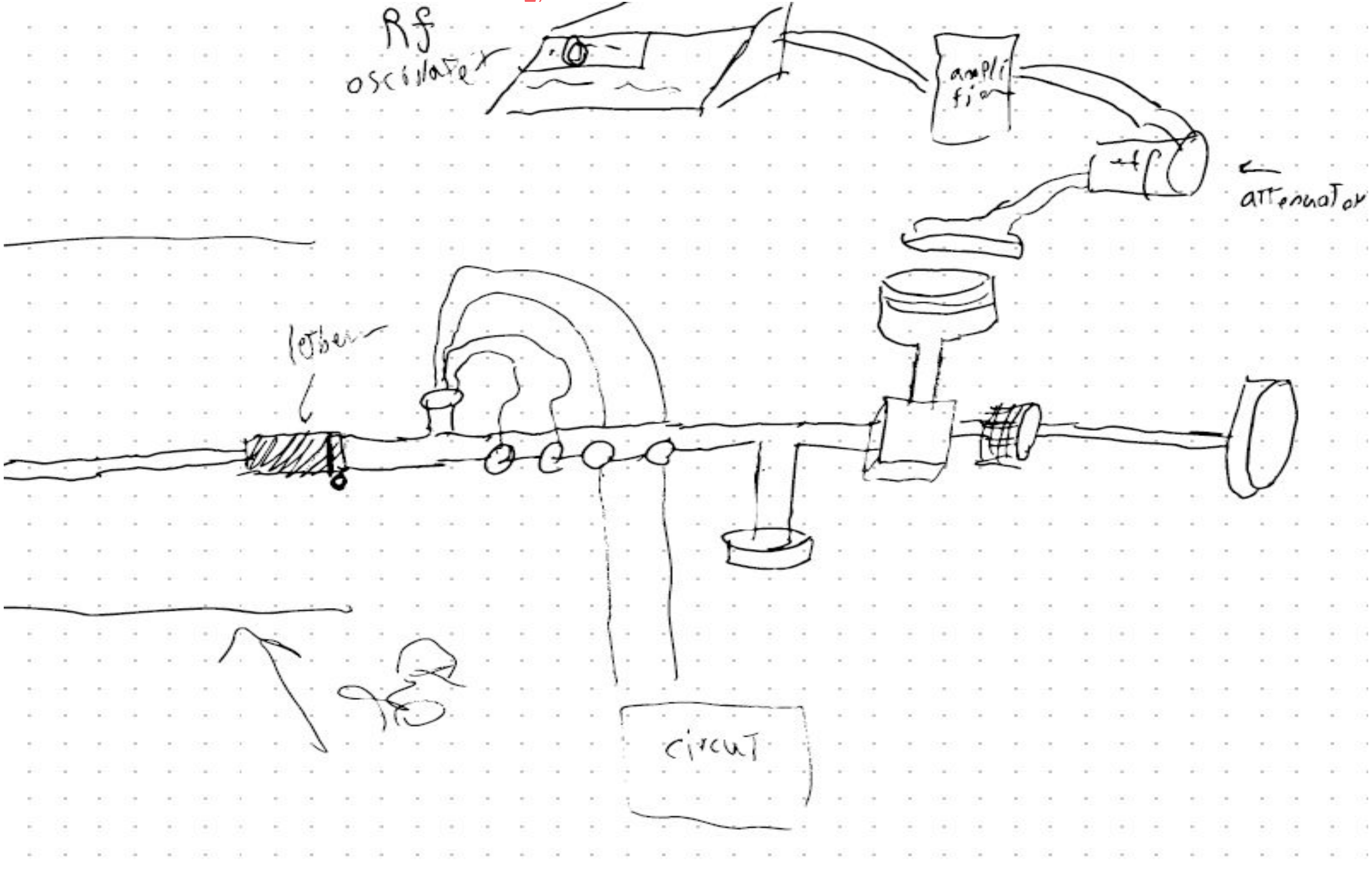


Setup the AC Effect!

Turning on the RF Oscillator and the Microwaves doubler power supply. Then measure the RF power. Setting
the attenuator to 20 dB, then attach the power meter to the waveguide like the picture

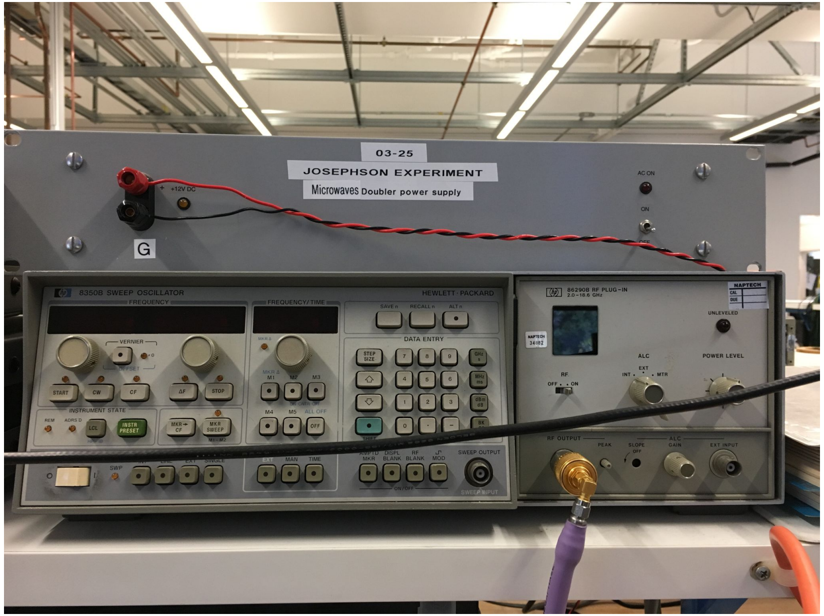
Note that we are not using the crystal detector's connection to the scope

Insert the probe into the Cryostat!



RF Frequency

We should set it around 11 GHz
But obviously it changed!
For us it was 10.416 GHz



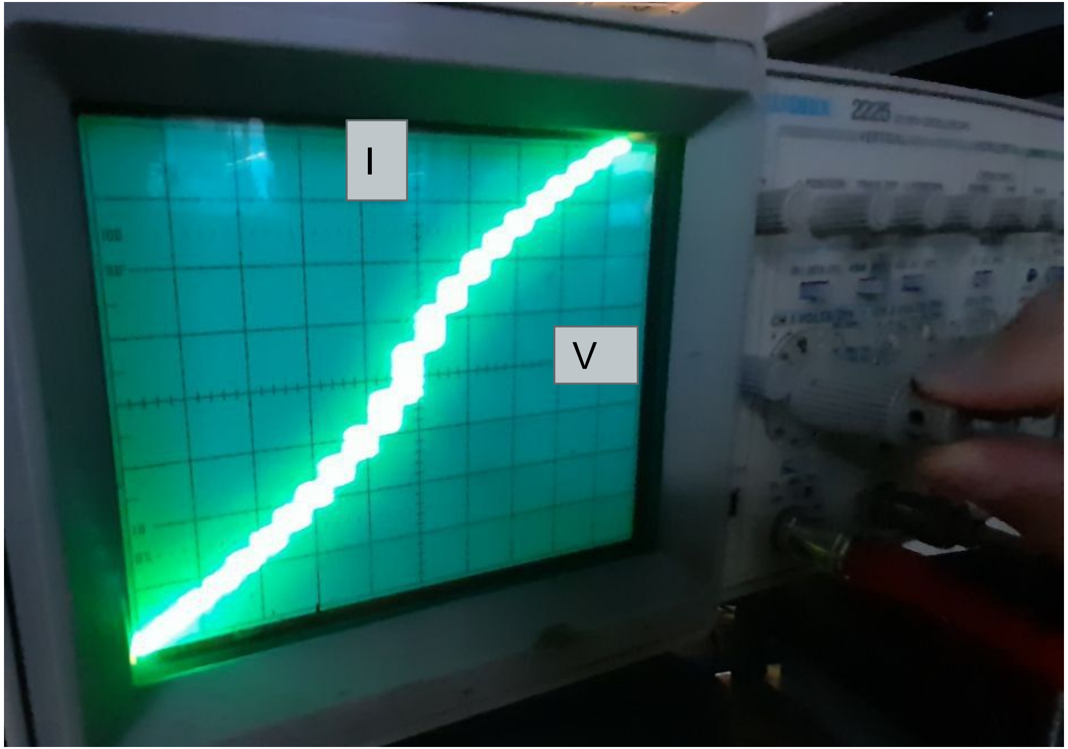
AC Effect!

On this we got 3 variables to change:

- RF frequency
- Rf power
- Junction’s pressure

Unlike the DC effect that we got only 1 variable and it was only 1 variable

We Get So Lucky!



Also, We made sure that is not a fake, and it was highly sensitive to magnetic fields!

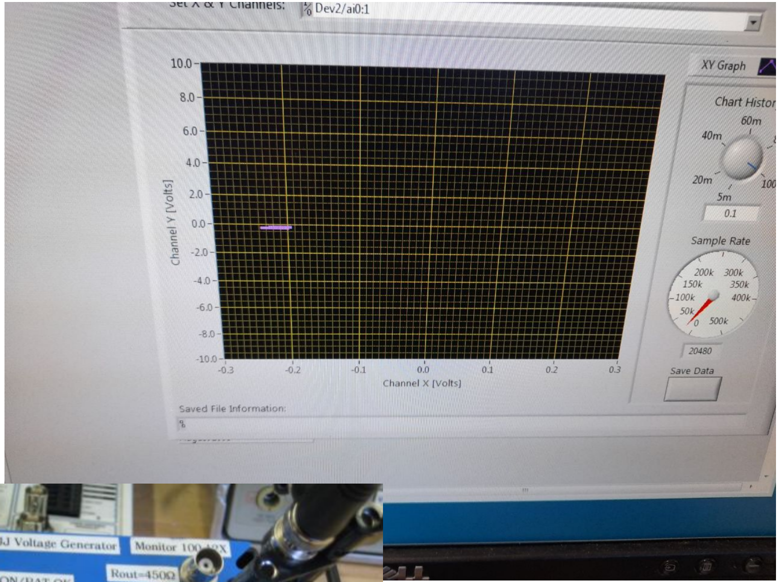
Calibration!

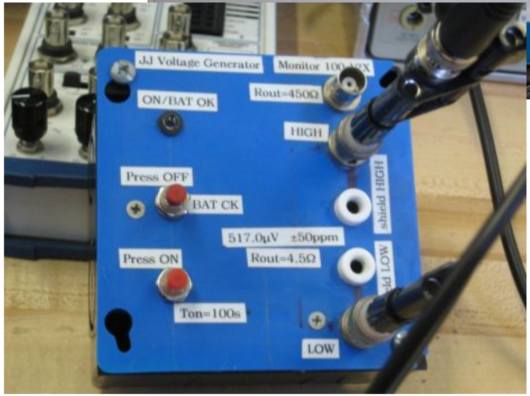
This box supplied 517uV.

We did the calibration by connecting the ends to DAQ.

Large error and it is mostly coming from the devices such as preamp. When it’s off the spread almost becomes a point!

More on this on Data analysis section!





Data Analysis

Calibration!

We measured the box voltage with voltmeter:

Box Voltage = 512 uV

Must have used the 517uV that is the exact value

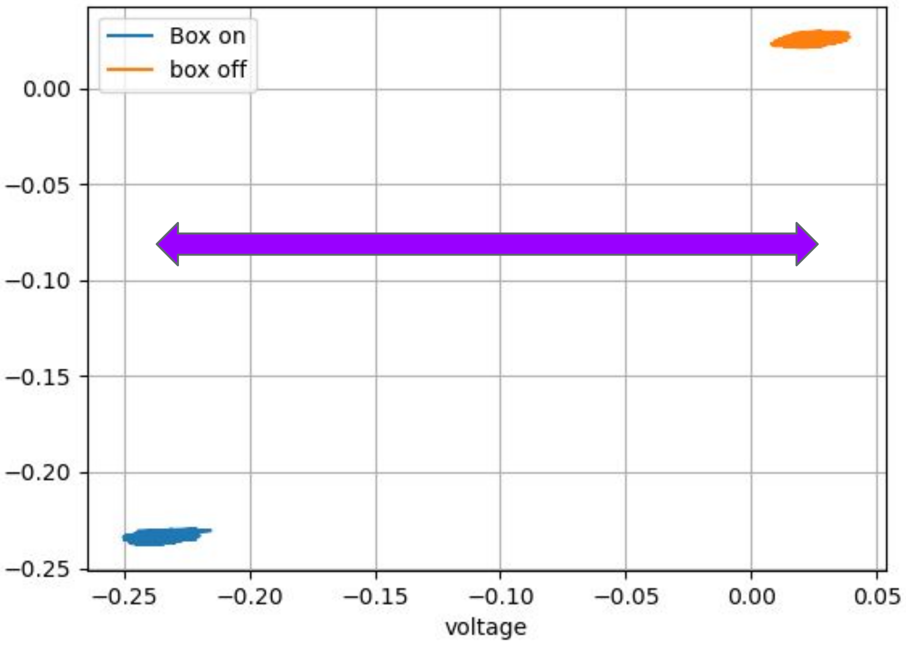
Box-on Average = - 0.2359 with std 0.0047

Box-off Average = 0.0232 with std 0.0045

Gain = |Box off Ave - Box On Ave| / (Box voltage)

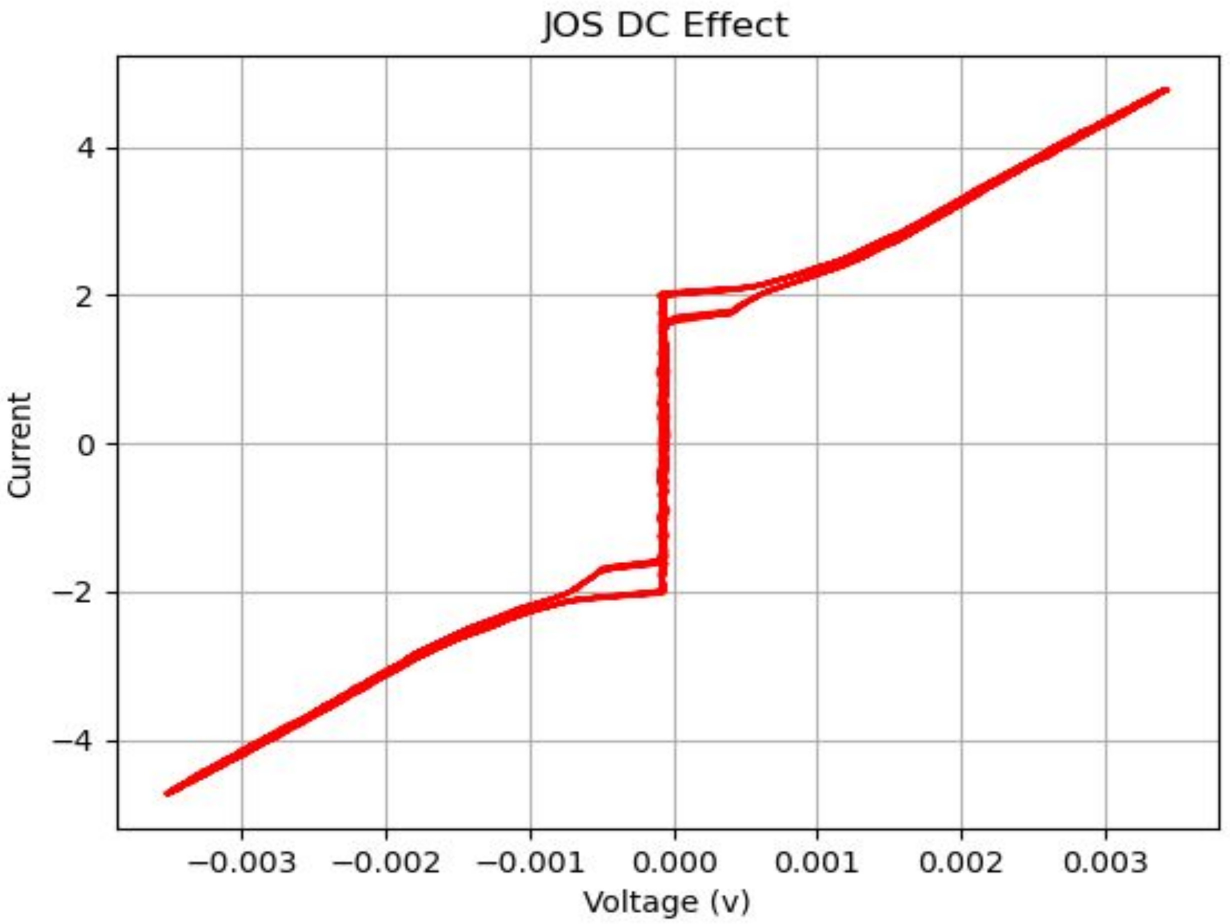
Gain = 505.50710 with std 0.0065 This finding is consistent with the gain we set on preamp (gain of 500)

V(calibrated in volts) = V(not calibrated) / Gain



Note: From now, all plots will be calibrated and in standard units!

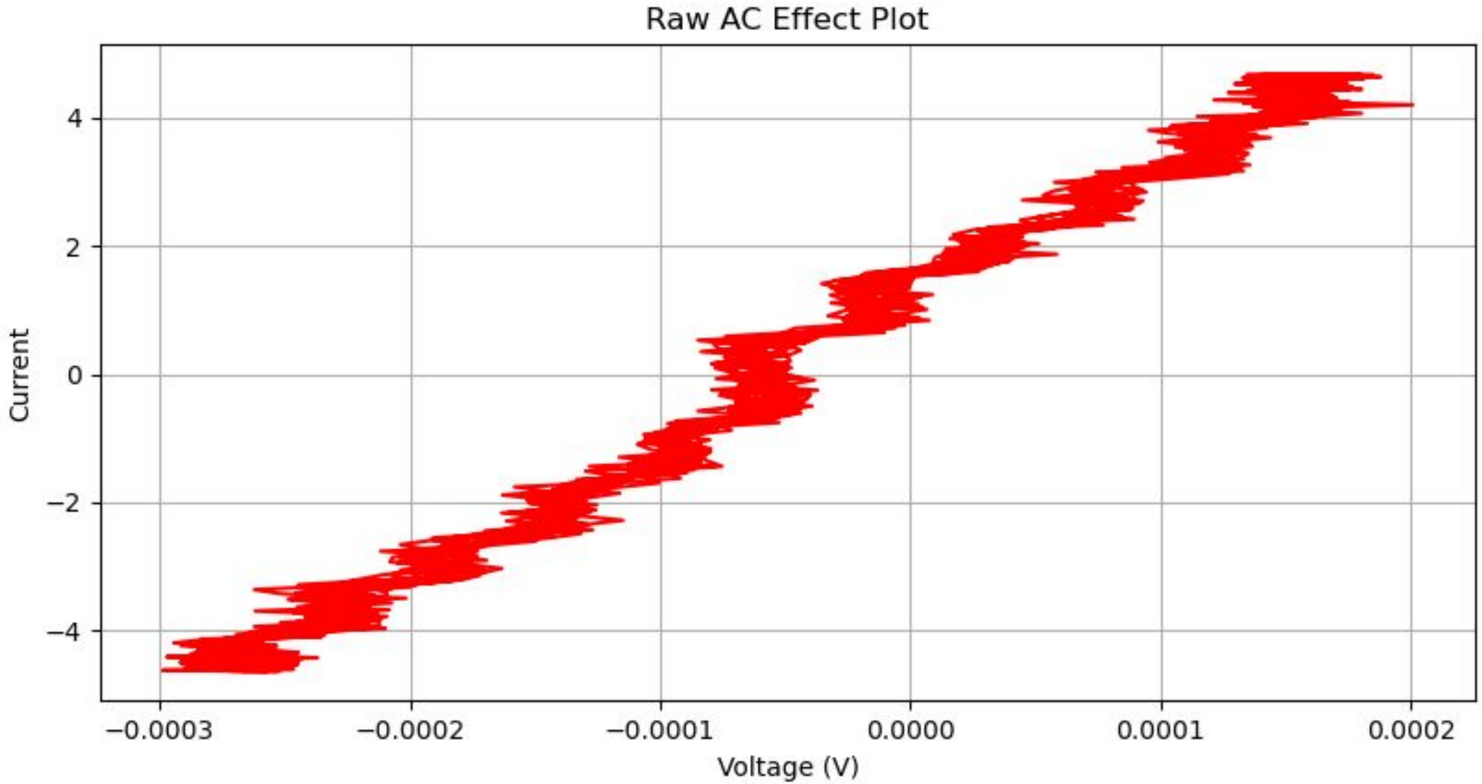
DC Effect!



JOS DC Effect
Current
Voltage (v)
4
2
0
−2
−4
−0.003
−0.002
−0.001
0.000
0.001
0.002
0.003

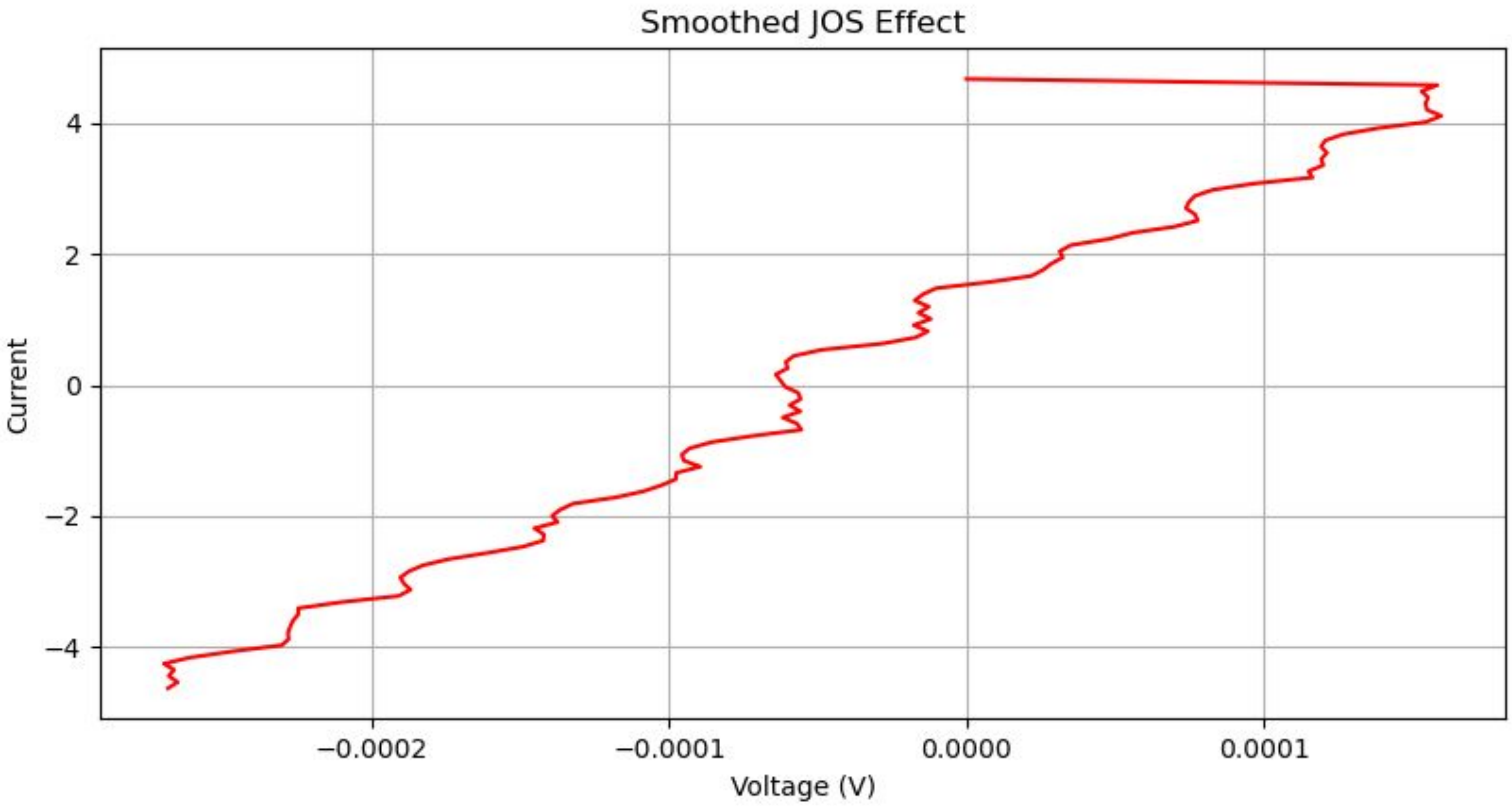
AC Effect

Noisy!



AC Effect

Smooth!



Used 100 current bins, grouped the voltages, then averaged the voltages within that bin!

AC Effect

Taking the absolute value of the numerical derivative .

Then convolving the function with a normalized array of 2 elements!

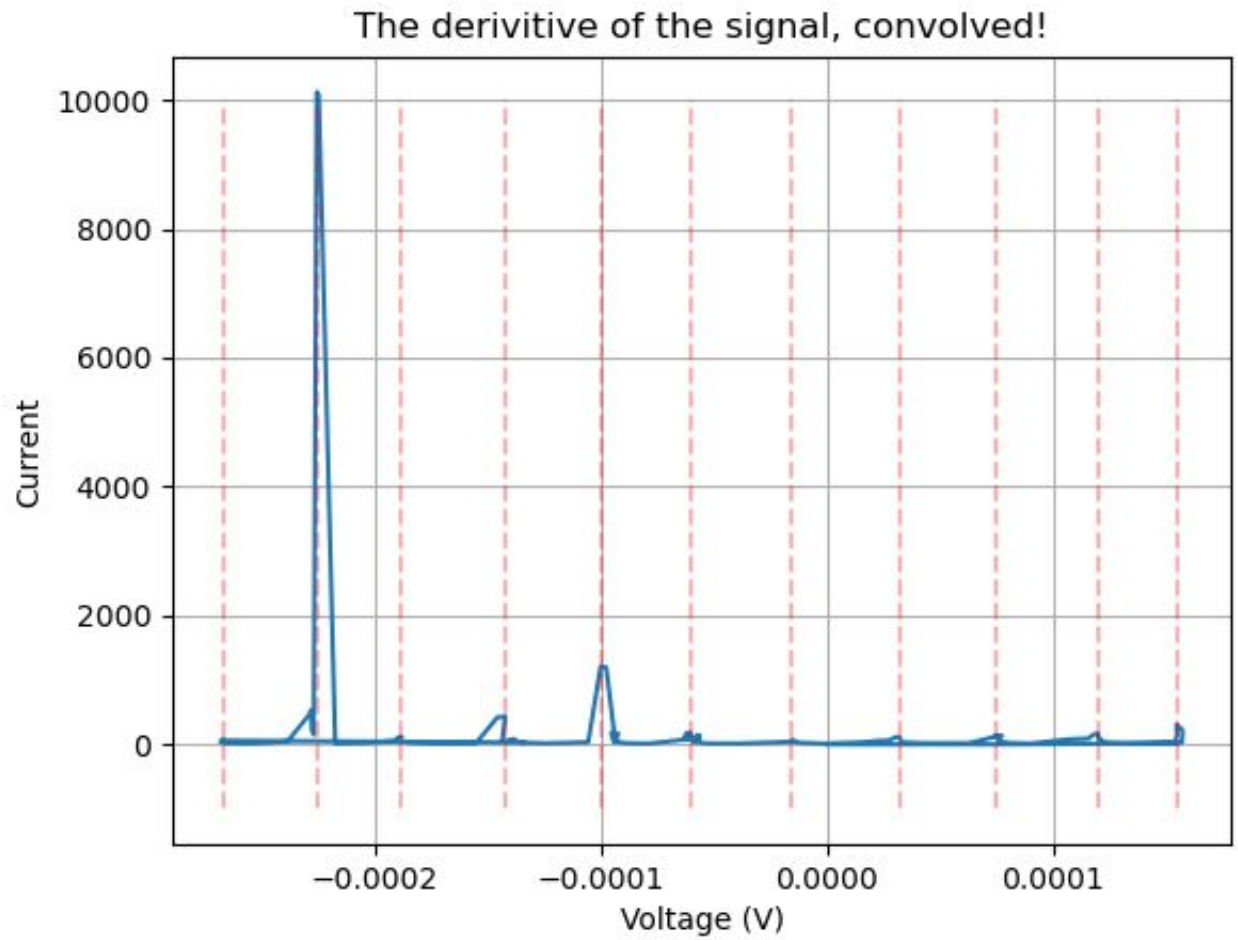
2 yields the best result!

Then we must find the peaks of all these spikes!

11 peaks, 11 maxima
Find the max signal within each range!

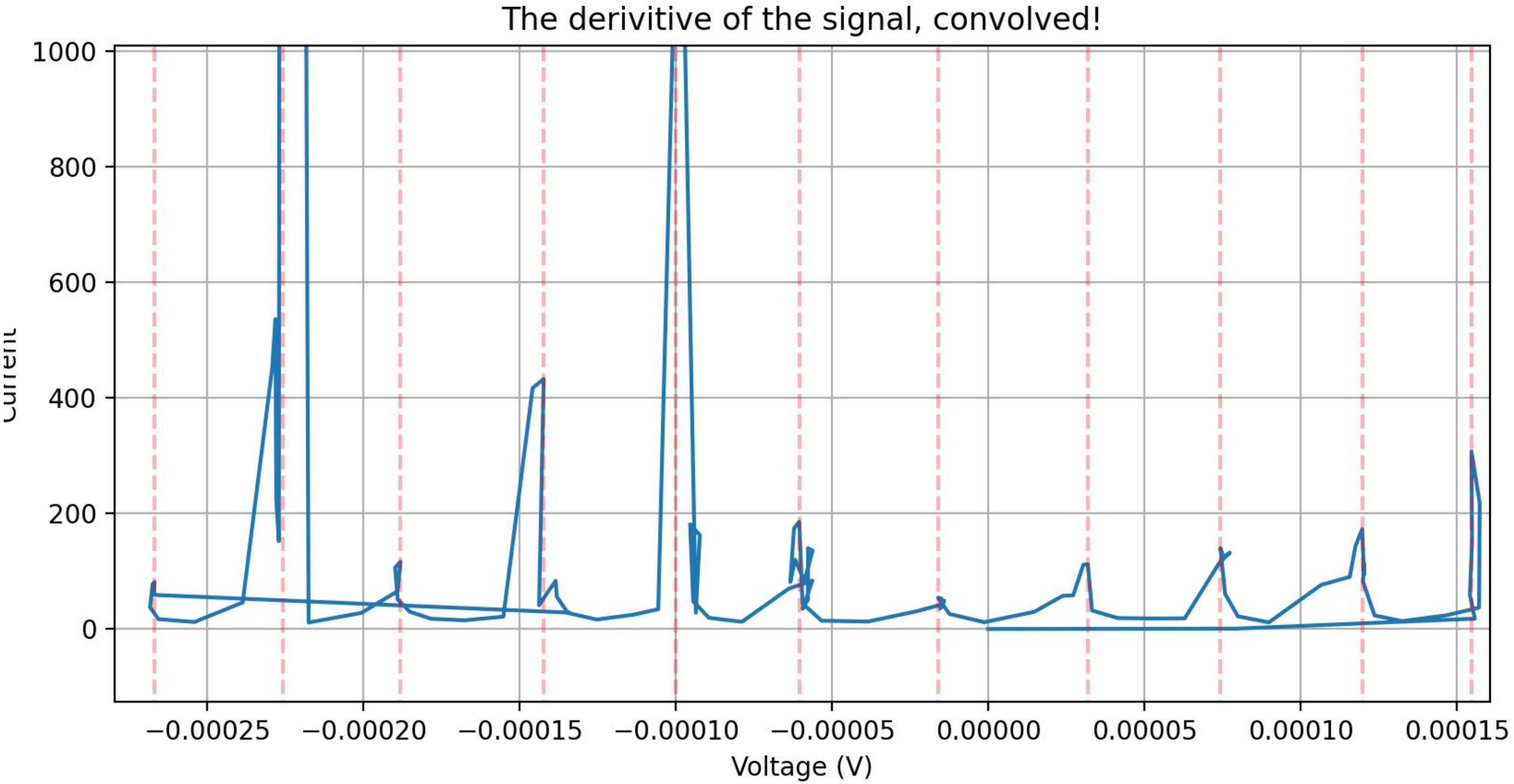
AC Effect

11 steps, 11 peaks!



AC Effect

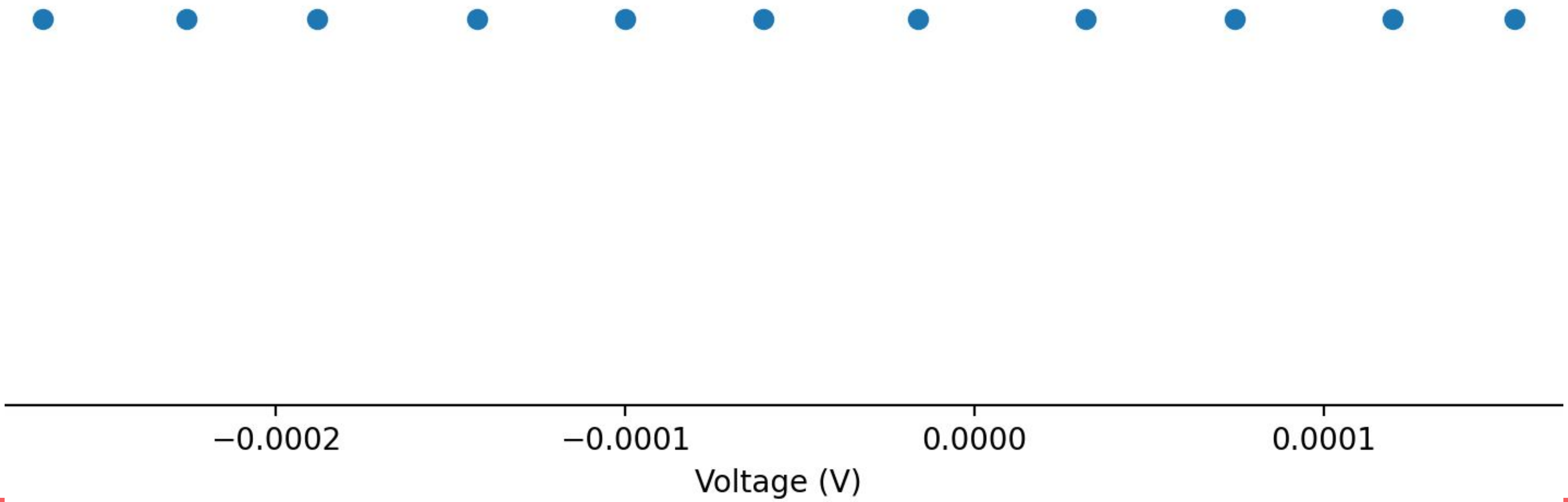
A Zoomed-in view



AC Effect

Visually...
This is really cool!
Just by eyes the distances seems really even and the same

Peak spacings representation on 1 dimension!



AC Effect

11 peaks,
10 differences between peaks

Differences from left to right!

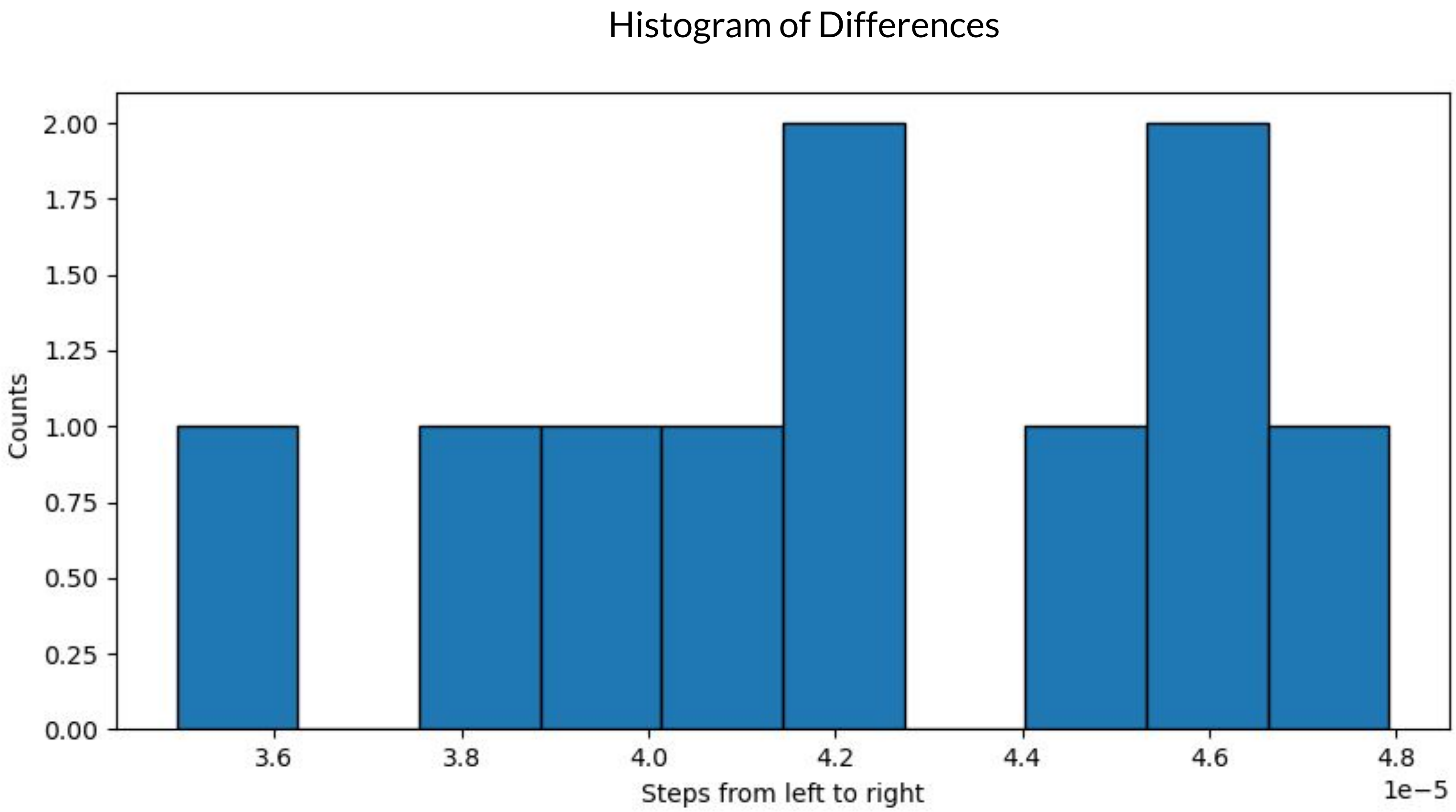
[4.1147e-5, 3.7557e-5, 4.5816e-5, 4.2126e-5, 3.9631e-5, 4.4385e-5, 4.7918e-5, 4.2413e-5,

4.5348e-5, 3.4952e-5]

the mean of differences is: 0.00004213 = 4.213e-5

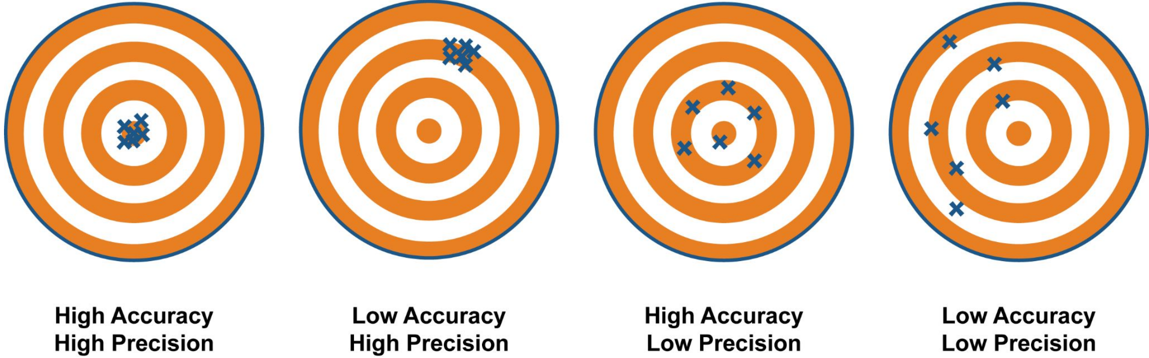
The error in differences is: 0.00000377 = 3.77e6

AC Effect



Histogram of Differences
Counts
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
3.6
3.8
4.0
4.2
4.4
4.6
4.8
Steps from left to right
1e−5

Measuring 2e/h




```
for difference 0 2e/h is:   506280054484484.75
for difference 1 2e/h is:   554673734800421.94
for difference 2 2e/h is:   454681873255395.4
for difference 3 2e/h is:   494514358854942.44
for difference 4 2e/h is:   525637114832805.7
for difference 5 2e/h is:   469339498030277.6
for difference 6 2e/h is:   434740081289784.06
for difference 7 2e/h is:   491163956938839.8
for difference 8 2e/h is:   459370733407445.2
for difference 9 2e/h is:   596008319422054.5
mean of the constants is:   498640972531645.1
std of the constants is:  4679381055470.323
```

std/sqrt(datapints = 11

Good accuracy

Good precision
Consistent data

Comparing with Established Values

Using the standard Planck's constant and Coulomb's charge:

h = 6.6e-34
e = 1.60217663e-19

→ 2e/h = 4.855e14

We get a percent difference of 2.7% comparing to the established value!!
This is pretty good!

Evaluation!

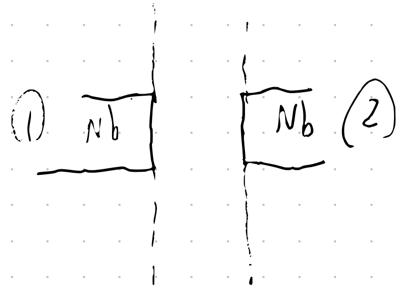
- Calibration: the accuracy of our results depends critically on the calibration the voltage axis! The acquired data was indeed so noisy! Most of it because of the preamplifier. That being on/off makes a huge difference!

- Also the Voltage data acquired with DAQ was noisy as well. We tried our best with a fine algorithm to reduce the effect, but still it effects the results

- In short Mostly systematic error, and not statistical error
- Includes instrumentional/Imperfection errors

Thank you for All Your Support!

Theory of Jos:



Consider each superconductor as a Q-system. for each we write schrö equation:

$$\begin{cases} i\hbar \frac{\partial}{\partial t} |\Phi_1\rangle = \hat{H} |\Phi_1\rangle \\ i\hbar \frac{\partial}{\partial t} |\Phi_2\rangle = \hat{H} |\Phi_2\rangle \end{cases}$$

where $\hat{H} = \hat{T} + \hat{U}$

$$\begin{cases} i\hbar \frac{\partial}{\partial t} |\Phi_1\rangle = \hat{T} |\Phi_1\rangle + U |\Phi_1\rangle \\ i\hbar \frac{\partial}{\partial t} |\Phi_2\rangle = \hat{T} |\Phi_2\rangle + U |\Phi_2\rangle \end{cases}$$

by Applying a Voltage of V across junction with the assumption that both ends are made out of same material & setting the middle point of two junction to be potential zero we say:

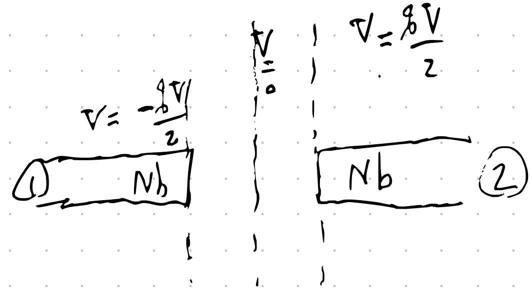
$$U = qV$$

where q is charge of a Cooper Pair!

Note that

$$\frac{qV}{2}|\Phi_1\rangle = \frac{qV}{2}|\Phi_2\rangle \quad \text{(I)}$$

sine both ends are same (Nb)



Hence:

$$\begin{cases} i\hbar \dfrac{\partial}{\partial t}|\Phi_1\rangle = \hat{T}|\Phi_1\rangle + \dfrac{qV}{2}|\Phi_2\rangle \\ i\hbar \dfrac{\partial}{\partial t}|\Phi_2\rangle = \hat{T}|\Phi_2\rangle - \dfrac{qV}{2}|\Phi_1\rangle \end{cases}$$

deliberatly did this according to (I)

The solns to these sets are:

$$\begin{cases} |\Phi_1\rangle = \sqrt{n_1}\, e^{i\phi_1(\vec{r},t)} \\ |\Phi_2\rangle = \sqrt{n_2}\, e^{i\phi_2(\vec{r},t)} \end{cases}$$

where ϕ_1 & ϕ_2 are phases

& $|n_1|$ & $|n_2|$ are the density of cooper pairs!

Plugging in the soln s & rearranging

The equations, we will have:

$$\begin{cases} -\frac{\hbar}{2}\frac{\partial n_1}{\partial t}\sin\phi_1 - \left(\hbar\frac{\partial\phi_1}{\partial t} + \frac{qV}{2}\right)n_1\cos\phi_1 = \hat{T}\sqrt{n_1 n_2}\cos\phi_2 \\ -\frac{\hbar}{2}\frac{\partial n_2}{\partial t}\cos\phi_1 - \left(\hbar\frac{\partial}{\partial t}\phi_1 + \frac{qV}{2}\right)n_1\sin\phi_1 = \hat{T}\sqrt{n_1 n_2}\sin\phi_2 \end{cases}$$

mult top by $\cos\phi_1$

bottom by $\sin\phi_1$

& then subtract them

We can get the time variation of charge densities & also the time variation equation of phases

for densities we get:

$$\begin{cases} \dfrac{\partial n_1}{\partial t} = \dfrac{2}{\hbar} \hat{T} \sqrt{n_1 n_2} \sin(\phi_2 - \phi_1) \\ \dfrac{\partial n_2}{\partial t} = -\dfrac{2}{\hbar} \hat{T} \sqrt{n_1 n_2} \sin(\phi_2 - \phi_1) \end{cases}$$

& for phases:

$$\begin{cases} \dfrac{\partial \phi_1}{\partial t} = \dfrac{-i}{\hbar} \hat{T} \sqrt{\dfrac{n_2}{n_1}} \cos(\phi_2 - \phi_1) - \dfrac{qV}{2\hbar} \\ \dfrac{\partial \phi_2}{\partial t} = \dfrac{-i}{\hbar} \hat{T} \sqrt{\dfrac{n_1}{n_2}} \cos(\phi_2 - \phi_1) + \dfrac{qV}{2\hbar} \end{cases}$$

Now fully considering $n_1 = n_2 = n$ identical superconductor

we have:

$$\begin{cases} \dfrac{\partial n}{\partial t} = \dfrac{\partial n_1}{\partial t} = \dfrac{\partial n_2}{\partial t} = \dfrac{2\hat{T}}{\hbar} n \sin(\phi_2 - \phi_1) & (II) \\ \hbar\left(\dfrac{\partial \phi_2}{\partial t} - \dfrac{\partial \phi_1}{\partial t}\right) = qV & (III) \end{cases}$$

These are the governing JOS effect equations

interestingly eq (II) suggests that even in an absence of a voltage, we have a current! (DC effect)

(II) represents the flow of Cooper Pairs across the gap

(III) gives us the rate of change of Phase!

⟹ Hence from III by integrating & setting $q = 2e^-$ (charge of pair)

we have:

$$\phi_2 - \phi_1 = \frac{2e^- V}{\hbar} t + \Delta\phi_0 \quad \text{IV}$$

initial Phase difference where is const of integration

Now from IV we can say:

$$\frac{\partial n}{\partial t} \propto I_J = \frac{2\hat{T}}{\hbar} n \sin\left(\omega_J t + \Delta\phi_0\right)$$

where $\omega_J = \frac{2e^- V}{\hbar}$

So here we need to Think...

$$I \propto \frac{2\hat{T}}{\hbar} n \sin\left(\left[\frac{2eV}{\hbar}\right] t + \Delta\phi_0\right) \quad (V)$$
J

This eq (V) is Josphson's current!

Even if no voltage is applied $(V = 0)$ There is still current flowing Through The junction!

~~A~~ let The total voltage across junction be The DC voltage plus The AC voltage induced by microwave field!

$$V = V_0 + V_1 \cos \omega_0 t$$

Plug in the voltage in equation (V) & solve for all parts (the kinetic operator & etc) we get

The most general form of current equation:

Complicated math!

$$I_J(t) = I_1 \sin\left(\frac{2eV_0}{\hbar} t + \frac{2eV_1}{\hbar\omega_0} \sin\omega_0 t + \alpha\right)$$

$$= I_1 \sum_{n=-\infty}^{\infty} \left\{ J_n\left(\frac{2eV_1}{\hbar\omega_0}\right) \sin\left[\left(n\omega_0 + \frac{2eV_0}{\hbar}\right) t + \alpha\right] \right\}$$

where I_1 is a const & J_n is Bessel function

& α is some const related to wave number

These results predict current spikes at $V = \frac{n\hbar\omega_0}{2e}$

and note that $\boxed{\omega_0 = \omega_J = \frac{2eV}{n\hbar}}$ where: $n = \ldots, -1, 0, 1, \ldots$

(i.e. Resonance frequency)

& getting back to main equation

we will use:

$$\omega_0 = \frac{2eV}{n\hbar} \Rightarrow \cancel{2\pi} f_0 = \frac{2eV(\cancel{2\pi})}{nh}$$

$$\Rightarrow f_0 = \frac{2eV}{nh} \Rightarrow \frac{nf_0}{V} = \frac{2e}{h}$$

in particular one of the spacings must be

$$\frac{f_0}{V} = \frac{2e}{h} = \text{const}$$

$$\frac{f_0}{V} = \text{Const}$$